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Gauge Principle for Ideal Fluids and Variational Principle

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Gauge-covariant variational formulation is given for flows of a compressible ideal fluid. Symmetry groups, i.e. the gauge groups, of fluid flows are a translation group and a rotation group. We propose a Galilei-invariant Lagrangian, and require its variation to be gauge-invariant (both global and local) with respect to the symmetry groups, and try to deduce the Euler's equation of motion for fluid flows from the gauge principle. The velocity field consistent with the first translation group is found to be irrotational, and corresponding equation of motion is that for potential flows. In complying with local gauge invariance with respect to the second gauge group, i.e. SO(3), a gauge-covariant derivative is defined by introducing a new gauge field. Galilei invariance of the covariant derivative requires that the gauge field coincides with the vorticity. As a result, the covariant derivative of velocity is found to be the material derivative of velocity. Thus, the Euler's equation of motion for an ideal fluid is derived from the Hamilton's principle. According to the Noether's theorem, the gauge symmetry with respect to the translation group results in the conservation law of total momentum, while the symmetry with respect to the rotation group results in the conservation of total angular momentum.

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