STREAMLINE TOPOLOGY OF THE NEAR WAKE OF A CIRCULAR CYLINDER AT LOW REYNOLDS NUMBERS

Morten Brøns*, Kristine Niss**, Bo Jakobsen**, Lars K. Voigt***, Rune Petersen*, Anders Bisgaard*

* *MAT, Technical University of Denmark, DK-2800 Kongens Lyngby, Denmark*
** *IMFUFA, Roskilde University, DK-4000 Roskilde, Denmark*
*** *HV Turbo, DK-3000 Helsingør, Denmark*

Summary Using methods from bifurcation theory, we find all possible instantaneous streamline patterns in the wake of a cylinder at Reynolds numbers close to the onset of vortex shedding and close to the cylinder. The analysis indicates that two different regimes are to be expected. Using numerical simulations, we verify the existence of these regimes.

The basic features of the flow around a circular cylinder of diameter D are well known. For low values of the Reynolds number $Re = UD/\nu$ the flow is attached to the cylinder. At $Re \approx 5$ the flow separates, and two counter-rotating steady vortices are created behind the cylinder. At $Re \approx 50$ the steady flow becomes unstable, and the vortices are shed periodically from the cylinder, resulting in the celebrated von Kármán vortex street. The transition to periodicity is a Hopf bifurcation, see e.g. [6]. For higher values of Re the flow becomes three-dimensional and turbulent.

The purpose of the present paper is to analyze the transition from steady flow to periodic flow on the basis of the streamline patterns. An attempt of such an analysis on the basis of numerous visualization experiments was performed by Perry *et al.* [5]. The result is shown in figure 1. The basic mechanism in the creation of new vortices is the merging of two dividing streamlines at the cylinder surface, as shown in the transition between panel (d) and (e) and again between (h) and (a). In this process, an attached vortex and a free stagnation point are created.

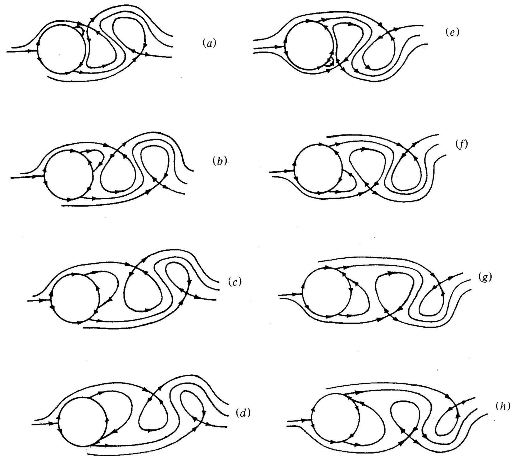

Figure 1: Instantaneous streamline pattern during a shedding cycle proposed by Perry et al. *[5].*

In the present paper we consider the equations for the instantaneous streamlines at time t_0, $\dot{\mathbf{x}} = \mathbf{v}(\mathbf{x}, t_0, Re)$, as a dynamical system, depending on a parameter Re, and, in the unsteady case, the time t_0. For certain values of the parameters the streamline pattern may be degenerate or structurally unstable, and arbitrarily small changes in the parameters give rise to qualitative changes in the streamline patterns. Using bifurcation theory, classification of the possible patterns close to a degenerate configuration may be obtained. For the present case, the flow close to a fixed wall with no-slip boundary conditions, the basic theory is established in [1] and [4]. Applications of this approach to the analysis of numerical simulations are given in e.g. [3].

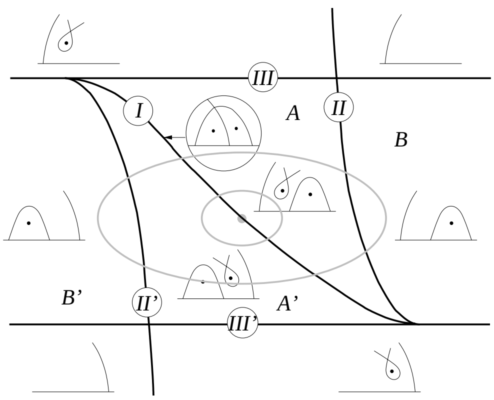

Figure 2: Bifurcation diagram. A slice in the parameter space is partitioned into regimes with different streamline patterns. In each region, the pattern is sketched. The horizontal lines correspond to the cylinder wall.

Taking the flow at the creation at $Re \approx 5$ of the two steady vortices as the degenerate configuration from which we perform the bifurcation analysis, we can prove the following

Theorem *Any velocity field which is a perturbation of the field at the creation of two symmetric vortices is locally equivalent to a member of the three-parameter family of velocity fields generated from the normal form streamfunction*

$$\psi = y^2(c_{0,2} + c_{1,2}x + c_{0,3}y + xy - x^3).$$

Here x denotes a coordinate along the cylinder surface, y is a coordinate orthogonal to the surface, and $c_{0,2}, c_{1,2}, c_{0,3}$ are free parameters.

The theorem is proved with methods from normal form theory. For proofs of similar theorems, see e.g. [2, 4]. The analysis of the normal form streamfunction results in a three-dimensional bifurcation diagram in the $c_{0,2}, c_{1,2}, c_{0,3}$ parameter space.

An interesting two-dimensional slice is shown in figure 2. The space is partitioned into regions *A*, *A'*, ... with different streamline topologies. The regions are bounded by bifurcation curves, shown heavy. The curve *I* is a global bifurcation curve. Here the streamlines has the topology of the steady vortex pair, but possibly with the symmetry broken.

To connect this result with the vortex shedding, we assume that the transition to periodic flow occurs at a Reynolds number so close to the creation of the steady vortices, that the flow patterns can be considered perturbations of the degenerate flow, and hence must be contained in the bifucation diagram. With this assumption, we can follow the development of the topology of the wake in the diagram as *R*eis increased. The steady state corresponds to the grey point on the global bifurcation curve *I*. As the flow becomes periodic, the temporal development of the streamfunction must be represented by a closed curve in the bifurcation diagram. Right after the bifurcation the amplitude is small, and is represented by the small ellipse in figure 2. Hence the theory predicts that right after the Hopf bifurcation, two different structurally stable streamline patters will prevail, namely *A* and its mirror image *A'*. At exactly two time instants during the cycle, the topology of the steady vortices will exist, as the bifurcation curve *I* is crossed.

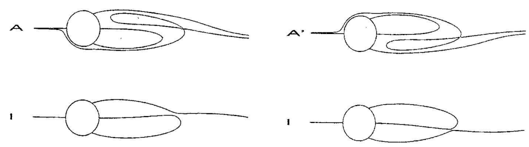

Figure 3: Numerical simulations for $Re = 45$, right after the Hopf bifurcation. The panels show instantaneous streamline patterns at different times during the shedding cycle.

Increasing *R*e, it is expected that the amplitude of the limit cycle will grow, resulting in a larger closed orbit in the bifurcation diagram. Hence, there is the possibility, that beyond a certain critical *R*e, further bifurcation curves can be crossed, as indicated by the large ellipse in figure 2.

The analysis above is of a purely qualitative nature, and relies only on the existence of a streamfunction for the flow. Hence, one cannot on this basis predict actual Reynolds numbers where transitions between different sequences occur. We have verified the scenarios by numerical simulations using a finite volume code. The code finds the Hopf bifurcation at $Re = 43$, in reasonable agreement with experiments. At $Re = 45$ a sequence of patterns corresponding to the small ellipsis in figure 2 is obtained. This sequence persists only until $Re \approx 45.8$, where the limit cycle intersects the bifurcation curves *II* and *II'*, and the streamline patterns *B* and *B'* appear during the cycle. No further changes are observed up to $Re = 200$.

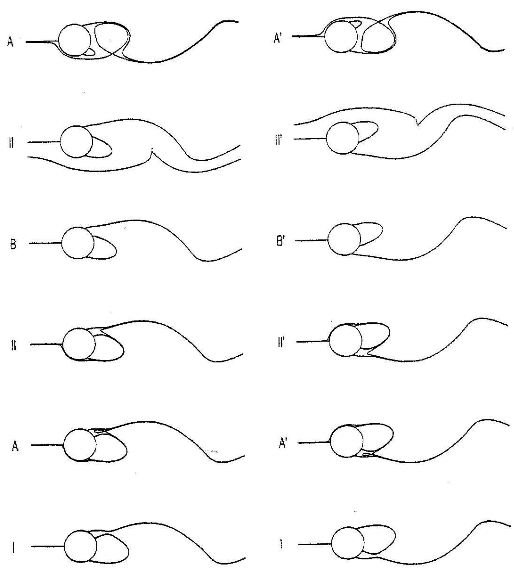

Figure 4: As figure 3, but for $Re = 100$.

Note, that the sequence in figure 1 is not obtained. It is a perfectly legitimate sequence, and Hartnack [4] shows that the sequence is in fact the simplest possible which can account for creation and destruction of attached vortices. However, it does not include the topology of the steady vortices, and the present analysis shows that this cannot be avoided generically.

The present qualitative analysis does not depend on the specific shape of the fixed body. For more general shapes such as ellipses the same bifurcation diagram will be relevant, as long as the streamline pattern of the steady vortex pair is of relevance.

References

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