

## DYNAMO ACTION IN STEADY HELICAL PIPE FLOW

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*Summary* The steady, pressure-driven flow of a conducting fluid down a helical pipe of rectangular cross-section is shown to drive a kinematic dynamo at moderate values of the magnetic Reynolds number,  $R_m$ . The asymptotic structure of the growing modes is analysed as  $R_m \rightarrow \infty$ . Numerical calculations are continued into the nonlinear regime.

### INTRODUCTION

A velocity field giving rise to spontaneous magnetic field generation in a conducting fluid is known as a *kinematic dynamo*. Various simple examples of kinematic dynamos are known, but there are very few dynamically self-consistent examples, in which the initial instability grows to a level where it interacts with the driving mechanism. In this paper we consider configurations with helical symmetry, as originally formulated by [1] and [2]. The Ponomarenko [3] kinematic dynamo can be regarded as a special case of helical flow. Here, a fully nonlinear dynamo driven merely by a steady pressure gradient along a pipe is described. The pipe has a helical shape with a rectangular cross-section, as sketched on the left in figure 1. The work has relevance to the construction of laboratory dynamos [4]. Details of the kinematic part of this paper can be found in [6].

### HELICALLY SYMMETRIC PIPE FLOW

Helical symmetry is a natural generalisation of two-dimensionality ( $\varepsilon = 0$ ) and axisymmetry ( $\varepsilon \rightarrow \infty$ .) In terms of cylindrical polar coordinates  $(r, \theta, z)$ , a scalar function is helically symmetric if it depends only on  $r$  and  $\phi = \theta + \varepsilon z$ , where  $\varepsilon$  is a constant. The symmetry direction is designated by the Beltrami vector field,  $\mathbf{H}$ , which is related to the unit coordinate vectors  $\mathbf{e}_\theta$  and  $\mathbf{e}_z$  by

$$\mathbf{H} = (\mathbf{e}_z - \varepsilon r \mathbf{e}_\theta) / h^2 \quad \text{where} \quad h = (1 + \varepsilon^2 r^2)^{1/2} \quad \text{and} \quad \nabla \wedge \mathbf{H} = -\frac{2\varepsilon}{h^2} \mathbf{H}.$$

The Navier-Stokes equations and the magnetic induction equation are invariant with respect to this symmetry, and thus helically symmetric solutions to both can be found. The helically symmetric incompressible flow  $\mathbf{u}$  can be conveniently represented by two scalar functions  $v(r, \phi)$  and  $\psi(r, \phi)$  as

$$\mathbf{u} = v\mathbf{H} + \mathbf{H} \wedge \nabla \psi$$

The governing equations for  $\psi$  and  $v$  are geometrically linked.

Steady, pressure-driven laminar flow down a helical pipe was calculated in [5] as a function of the hydrodynamic Reynolds number,  $R_e$ . A solution is shown in figure 1 for an intermediate value of  $R_e$ . The left-hand diagram shows the contours of the cross-pipe flow. An important feature of the flow is that the cross-pipe flow has a stagnation point structure. This is found to be important for dynamo action.

### DYNAMO ACTION

Magnetic fields with the same helical symmetry as the flow are sought. The field  $\mathbf{B}$  is decomposed as

$$\mathbf{B} = (B\mathbf{H} + \mathbf{H} \wedge \nabla \chi) e^{\lambda t}$$

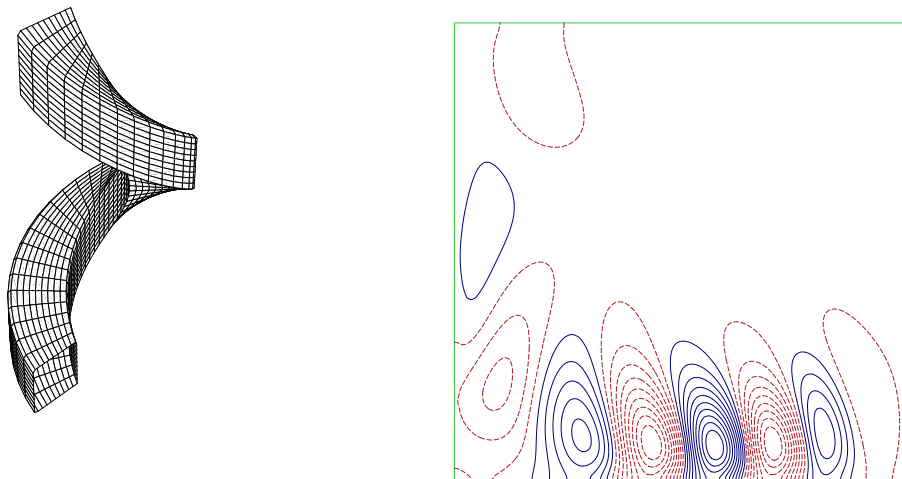
and the complex growth rate  $\lambda$  determined. Two cases are considered; firstly when the medium outside the pipe is perfectly conducting, and secondly when it is insulating. The latter more relevant to the construction of practical devices.

#### Perfectly conducting walls

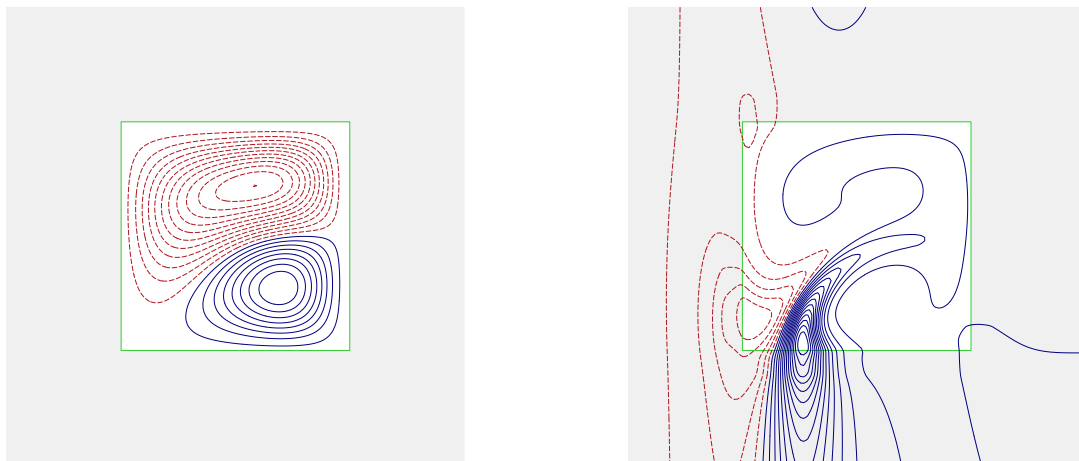
In this case the magnetic field is zero outside of the pipe region. This has a constraining effect on the dynamo. The pressure-driven pipe-flows do not in fact drive a dynamo with conducting walls, except in the limit as  $R_e \rightarrow \infty$ . This is because the cross-pipe flow is too strong. If it is artificially set to zero, a kinematic dynamo with rapid tangential variation is found as shown on the right in figure 1. The asymptotic structure of the modes as  $R_m \rightarrow \infty$  is analysed.

#### Insulating exterior

When  $\mathbf{B}$  is permitted to diffuse out of the pipe, a dynamo is found for  $R_m \sim 100$ . In figure 2 the growing downpipe component  $B$  is shown alongside the cross-pipe flow,  $\psi$ . It is clear that the stretching of the field at the separation line is an important feature of the dynamo process. The other vital ingredient is the manner in which helical symmetry includes



**Figure 1.** Left: A helical rectangular pipe for  $\varepsilon = 1$ . Right: a kinematic dynamo for a velocity field with  $\psi = 0$  and conducting walls.



**Figure 2.** Left: Cross-pipe flow ( $\psi$ ) at  $R_e = 20^3 h_b$ . At this  $R_e$  the secondary flow has a separation point. Right: the growing  $\mathbf{H}$ -component of the field for  $R_m = 15^3$  and weak external conductivity. The effect of field-stretching near the separation point is pronounced. Only part of the exterior is shown.

a term involving  $B$  in the  $\chi$ -part of the induction equation, a “geometrical  $\alpha$ -effect.” The field in figure 2 was actually calculated with a small exterior conductivity, but is almost a potential field.

Computations of the time evolution and saturation of the dynamos described here are currently being performed, and results will be presented at the meeting.

## References

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