

Subgrid-scale modeling of anisotropic MHD turbulence

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Application of electromagnetic technologies to materials processing often involves interaction between a steady magnetic field and a turbulent flow of an electrically conducting fluid. Prominent examples are continuous casting of steel, growth of semiconductor crystals, ‘direct chill’ casting of aluminum and primary aluminum production in Hall-Herault electrolysis cells. The hydrodynamic aspects of this phenomenon have been a focus of interest of hydrodynamic community for several decades (see, e.g., Moreau 1990 or Davidson 2001 for a review). Nevertheless, our understanding of fundamental properties of MHD turbulent flows is not only far from complete but falls considerably behind the level of current understanding of “classical” non-magnetic turbulence. Accurate laboratory experiments with liquid metals or oxide melts are very difficult and, often, prohibitively expensive to perform. Therefore, numerical simulation becomes a virtually indispensable tool for studying such flows.

We consider the case typical for practically all the industrially relevant flows, which is characterized by large values of hydrodynamic Reynolds number but small values of magnetic Reynolds and Prandtl numbers. Direct numerical simulation of such flows is hardly feasible. On the other hand, the RANS (Reynolds averaged Navier-Stokes) method used by commercial CFD programs often does not provide desired accuracy even after special adjustment of the model accounting for MHD effects (Widlund et al 1998, Kenjereš & Hanjalić 2000). In the non-magnetic case, significant advancement in the simulation of turbulence has been achieved in the last 3 decades through development of LES (large-eddy simulation) approach that combines the accuracy of DNS and computational efficiency of RANS. In LES, a space-filtered velocity field corresponding to fluid motions at large and moderate (energetic and inertial range) length scales is directly calculated on a fine computational grid. The effect of subgrid-scale fluctuations is modeled using one of the subgrid-scale (SGS) closure hypotheses.

The fundamental properties of MHD turbulence were subject of experimental, numerical and theoretical investigations (see, e.g., Moffatt 1967, Alemany et al 1976, Davidson 1997, Zikanov & Thess 1998). It was found that, at sufficiently large Stuart number $N \equiv \sigma B^2 L \rho^{-1} U^{-1}$, the properties of turbulent fluctuations change dramatically. The action of the Lorentz force has a dissipative character, the corresponding term in the turbulent kinetic energy equation being, of course, the Joule dissipation term, which is anisotropic. The dissipation rate of kinetic energy is proportional to $\cos \theta^2$, where θ is the angle between the wavenumber vector of a Fourier flow mode and the imposed magnetic field. This means zero dissipation of modes independent of the coordinate along the magnetic field lines and maximum dissipation of modes with $\mathbf{k} \parallel \mathbf{B}$. The magnetic field tends to eliminate velocity gradients and to elongate the flow structures along the magnetic field lines. The flow becomes anisotropic or even two-dimensional. This tendency toward two-dimensionality is opposed by the non-linear energy transfer between the flow modes. The interplay between the Joule dissipation and the non-linear interaction leads to complex non-linear flow evolution. In particular, this evolution may include time- and spatially intermittent behavior (Zikanov & Thess 1998).

Transformation of turbulent flow field into anisotropic and intermittent form means that the existing SGS closures developed for locally isotropic flows can not be assumed a-priori suitable for MHD turbulence. More careful consideration, adaptation of existing models and, perhaps, development of a new model is necessary. A first attempt was undertaken by Knaepen & Moin (2003) who applied the standard dynamic Smagorinsky model to the case of decaying homogeneous MHD turbulence at moderate Reynolds number. The model showed good accuracy, which can be explained by the fact, that, for such flows, the anisotropy is virtually scale-independent in the inertial range and can be accounted for through the self-adjusting mechanism of the dynamic model. At higher Reynolds number, one can, however, expect scale-dependence of the anisotropy and, thus, necessity of specially designed SGS model.

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The purpose of our work is to investigate the fundamental properties of turbulent fluctuations in anisotropic MHD turbulence and to develop an SGS closure capable of accurate and physically adequate representation of the flow. We perform a series of high-resolution DNS simulations of a flow in a three-dimensional rectangular domain with periodic boundary conditions. A uniform steady magnetic field is imposed in the vertical direction. Since the case of small magnetic Reynolds and Prandtl numbers is considered, we apply the quasi-static approximation. Induced perturbations of the magnetic field are neglected in comparison with the imposed magnetic field and the rotational part of the Lorentz force is reduced to a linear functional of velocity. The flow is forced at its largest scales in such a way as to keep the energy of flow modes at these scales at a constant level. Pseudo-spectral FFT-based numerical method is applied. The numerical resolution is $256^2 \times 512$ or $512^2 \times 1024$ spectral modes. We use the DNS data to study the properties of turbulent flow field and to perform a-priori analysis of possible SGS closures.

The work is supported the US Department of Energy, grant DE-FG02-03ER46062. The computations are performed at the University of Michigan – Dearborn using the parallel workstation cluster acquired with support of the US National Science Foundation, grant MRI CTS0320621. The authors are thankful to Dr. B. Knaepen for useful discussions and help with pseudo-spectral code.

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