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NUMERICAL SIMULATIONS OF DYNAMO EXPERIMENTS

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Summary Several experimental groups throughout the world have reproduced the dynamo effect on a laboratory scale or are preparing to do so. All experiments rely on numerical simulations, both in the design phase and for data analysis. This talk presents a few examples of how experiments and simulations interact, and where they agree and disagree, in particular for the Karlsruhe experiment. Possible implications for the geodynamo will also be discussed.

An experiment built in Karlsruhe has recently reproduced the dynamo effect on a laboratory scale . In this experiment, liquid sodium is pumped through a vessel which contains 52 cells arranged in a square lattice. Each cell consists of two coaxial pipes and a blade of helical shape inserted in the space between the inner and the outer pipe. The sodium flows straight through the inner pipe but is forced to flow along a right handed helix in between the two pipes of each cell. The flow alternates directions between adjacent cells. Two different length scales characterize the experiment: The size of each cell and the size of the entire sodium vessel. The two length scales differ by a factor somewhat larger than 8. This approximate separation of scales sets the Karlsruhe experiment apart from another successful experiment run in Riga.

In the experiment, the 52 cells are enclosed in a cylinder. However, parts of the piping which lead sodium from one cell to a neighboring cell stick out from the cylindrical container. The real geometry is too complicated to be simulated exactly.

A dynamo in a finite cylinder is cumbersome to handle numerically because of the corners and because there is no analytic expression for a potential field outside a finite cylinder. For the simulations, the cylinder has been embedded in the smallest sphere capable of enclosing the cylinder. The space between the cylinder and the spherical boundary is assumed to be filled with sodium at rest. This surrounding sodium is of course the strongest approximation involved in the simulations presented here. In accompanying computations, the space between cylinder and sphere has been filled with material of magnetic diffusivity 10 times larger than the diffusivity of sodium. These computations have shown that the outer conductor reduces the critical volumetric flow rates and that the general structure of the magnetic field is not affected by the outer conductor.

Another uncertainty concerns the value of the magnetic diffusivity. The simulations use a uniform diffusivity whereas the experiment consists of both sodium and stainless steel structures. Moreover, the spatially averaged resistivity is anisotropic. The resistivities perpendicular and parallel to the cell axis are a factor 1.3 and 1.06 larger than the resistivity of pure sodium, respectively. Turbulence further increases the magnetic diffusivity. However, the turbulent diffusivity can be estimated at $2.5 \times 10^{-3} m^2/s$ which is a negligible contribution compared with the molecular value.

Despite all the approximations involved in the simulations, the central quantities are identical in the experiment and the simulations. The onset of dynamo action and the field morphology outside the sodium are well predicted numerically. Discrepancies still exist in the shape of the interior field and some aspects of the dynamics of the magnetic field are not yet understood.

Peffley et al. have started an experimental investigation of a class of flows originally studied numerically by Dudley and James. Similar flows are currently under study by other groups as well. These flows exist in a sphere and are axisymmetric. Two counterrotating propellers with coaxial shafts set the liquid sodium into motion. The magnetic Reynolds number Rm reached in the experiment is insufficient to obtain self-excitation of the magnetic field $\mathbf{B}$. The approach to the critical Rm is diagnosed by applying an external magnetic field either parallel or perpendicular to the propeller axes and by observing the decay of $\mathbf{B}$ after the external field has been switched off. The radial component of the magnetic field is measured by an array of Hall probes placed on a ring coaxial with the axis of symmetry of the flow. The decay rates of the non-axisymmetric fields excited by an external field perpendicular to the axis of symmetry increase with increasing Rm. On the other hand, a field parallel to the shafts excites predominantly axisymmetric modes. The decay rate of these modes decreases with increasing Rm up to the highest Rm reached in the experiment. Extrapolation of the experimental curve to higher Rm predicts that the growth rate will become positive at sufficiently high Rm (around 160). Peffley et al. conclude that non-axisymmetric components in their flow must be essential to the dynamo mechanism because an axisymmetric velocity field generates a dynamo field without any axisymmetric component according to kinematic dynamo theory.

The second experimental finding of relevance here is the apparent spatial variability of the decay rates. Surprisingly, different decay rates of the magnetic field are measured at different locations. It is not surprising that different decay rates are measured from pulse to pulse if the external magnetic pulse is repeatedly applied to a turbulent flow. However, if the decaying field following a single pulse corresponds to the eigenmode with the smallest decay rate, one expects to find one single decay rate at all positions after an initial transient. The value of that decay rate depends on the actual shape of the (time dependent) velocity field during the decay.

It will be shown in this talk that the experimental data are nonetheless compatible with the assumption that global magnetic eigenmodes of an axisymmetric velocity field are observed. A model based on kinematic dynamo theory reproduces the experimental findings if one assumes that a superposition of eigenmodes is excited by the combined effect of the external field and turbulent fluctuations.