OPTIMAL LIFT FORCE ON VESICLES NEAR A DEFORMABLE SUBSTRATE

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Summary We investigate the effects of a deformable substrate on the dynamics of a vesicle in a shear flow. This system can be viewed as a zeroth order model for studying the complex coupling between blood cells and the glycocalyx layer which covers the internal part of microvessels. More precisely, we show that the lift force on spherical vesicles (a model for the leucocytes) exhibits a maximum for a rigidity of the substrate which lies in the physiological range. In the case of unswelled vesicles (for red blood cells), the maximum tends to disappear and the lift force is constant below this value, which appears now as a limit between two different regimes.

Understanding blood rheology is a formidable task in general, essentially due to the biphasic nature of this fluid. Various cells enter into the composition of blood, but the most important from the mechanical point of view are probably the red blood cells (RBC) and the white blood cells, or leucocytes. Typically, RBC represent around 45 % of the blood volume. These cells are evolving in a complex system of blood vessels, from the macroscopic veins and arteries to the microscopic veinules. In the first case, the polyphasic structure of blood can be neglected due to the huge scale separation between the typical size of the vessels (around 1 cm) and the cells size (around 10 μm). The dynamics of blood in this regime is dominated by inertia and turbulent flows can even be observed [1]. In this paper, we would like to restrict ourself to the opposite limit, when the typical size of cells can not be neglected anymore. It is known that this limit is associated with a variation of the apparent viscosity of blood with the diameter of the vessel: this is the well known Fahraeus-Lindquist effect.

Moreover, it is known that the internal part of this blood vessels is covered with a deformable glycocalyx layer, consisting of a brush-like structure made of bound and adsorbed macromolecules (proteoglycan, hyaluronan and unsulfated glycosaminoglycan). This layer may play an important role in the exchange-processes with the surrounding cells. From a mechanical point of view, the influence of this layer has been investigated by several authors, who modelized it as a porous medium, in which the fluid dynamics is governed by a Brinckman equation: $\eta\left(\nabla^2 - \frac{1}{\Sigma_p}\right)\boldsymbol{v} = \boldsymbol{\nabla} P$, with η the viscosity and Σ_p the Darcy permeability of the layer. Recent works [2] include the possible variation of this parameter with the compression of the layer. These studies were essentially concerned with the measurement of the hematocrit or the modification of the Fahraeus-Lindquist effect, and were performed in symmetrical geometry. The question of the individual coupling between the dynamics of cells and this layer remains an open question.

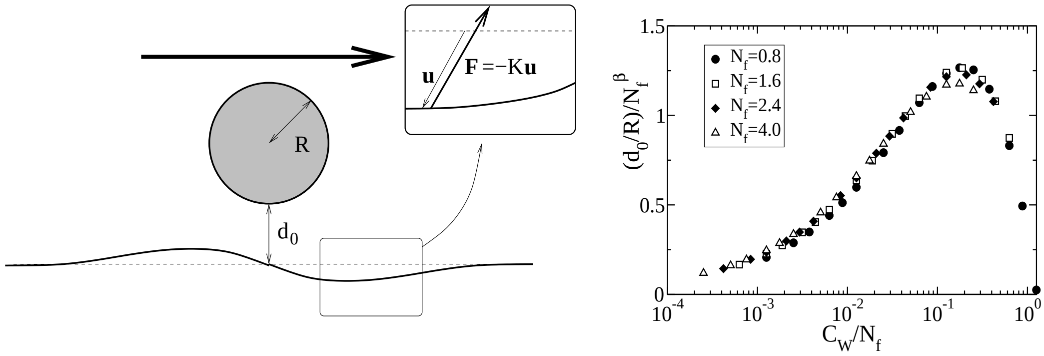

FIG. 1. Sketch of the system under consideration (left) and equilibrium height of the particle as a function of the rigidity of the substrate (right).

Previous numerical [3,4] or experimental [5] works have shown that the presence of a rigid wall near a vesicle (which can be seen as a good model for cells) induces a repulsive force. The fundamental origin of this force is clear: let us first consider a sphere near a rigid substrate, and submitted to a linear shear flow. The symmetry upon time reversal of the Stokes equation implies the absence of any lift force. This symmetry will be broken if we replace the sphere by a vesicle (or any deformable object) and a lift force is then expected. This lift force will be clearly modified if the substrate itself becomes deformable. For the sake of simplicity we modelized the deformable layer as a simple elastic medium: the force $\boldsymbol{f_{wall}}$ exerted on the surrounding fluid is simply proportional to the local displacement $\boldsymbol{u}$ (see figure 1 (left)):

$$\boldsymbol{f_{wall}} = -K\boldsymbol{u}. \tag{1}$$

K can be related to the Young modulus E of the glycocalyx layer by the relation $K \sim E/R$ where R is the size of the particle, which is the only length scale in the problem. The dynamics of the fluid will be described by the Stokes equation due to the vanishing Reynolds number (typically of order 10^{-2}-10^{-3} in this case):

$$\eta \nabla^2 \boldsymbol{v} - \boldsymbol{\nabla} P + \boldsymbol{f_{wall}} + \boldsymbol{f_F} + \boldsymbol{f_{mem}} = \boldsymbol{0} \tag{2}$$

and the appropriated no-slip boundary conditions at the surface of the vesicle and on the substrate. Moreover, we introduce a confining constant force $\boldsymbol{f_F}$: starting from an arbitrary position, the particle will progressively reach a stationnary state, when the lift force is compensated by this force. $\boldsymbol{f_{mem}}$ is the local force applied by the vesicle to the flow. We will restrict ourself to the 2D case, essentially for numerical reasons. $\boldsymbol{f_{mem}}$ can thus be obtained by a functional derivation of the 2D Helfrich free energy $E_H = \int_{mem} (\frac{\kappa}{2} c^2 + \omega) ds$, with κ and c respectively the bending energy and the local curvature of the membrane; ω is a Lagrange parameter enforcing constant perimeter. It can be shown that [3]:

$$\boldsymbol{f_{mem}} = \left\{ -\kappa \left[\frac{c^3}{2} + \frac{d^2 c}{ds^2} \right] + \omega c \right\} \boldsymbol{n} + \frac{d\omega}{ds} \boldsymbol{t} \tag{3}$$

with $\boldsymbol{n}$ and $\boldsymbol{t}$ the normal and tangential unit vectors at the membrane. The system is then entirely determined by only 4 parameters: $C_v = \eta\gamma R^3/\kappa$ characterising the deformability of the vesicle in the flow, $c_w = \eta\gamma/E$ for the deformability of the substrate, $N_f = \eta\gamma/\pi R f_F$ for the compensating force and the swelling ratio of the vesicle $\tau = 4\pi S/P^2$ (S being the surface and P the perimeter of the membrane (in 2D)), $\tau = 1$ for a sphere and < 1 for unswelled vesicles.

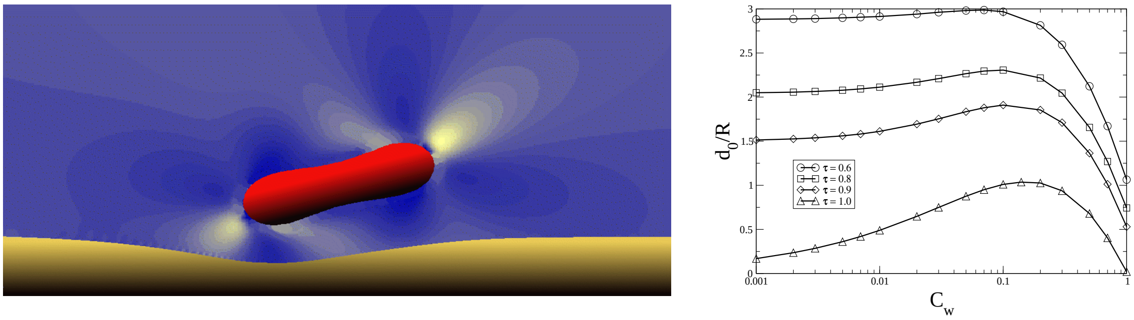

FIG. 2. Typical configuration of the system: color map represent the viscous dissipation rate (left) and equilibrium height as a function of the rigidity of the substrate for different swelling ratio (right).

Setting first $\tau = 1$ (which corresponds to circulating leucocytes), we have been able to show (using a Green function approach) that the equilibrium height of the particle, and so the lift force, exhibits a maximum for a given ratio C_w/N_f, as can be seen on the master curve (see figure 1, right). A lubrication analysis performed in the limit of quasi-rigid substrate allowed us to determine this scaling. In the case of vesicles we see in figure 2 that the maximum subsists even for swelling ratios of order 0.6, which is typical for RBC. Finally, we checked that the position of this maximum is independent on the swelling ratio (see figure 2 (right)) and on the deformability of the vesicle C_v in the range $0.1 < C_v < 100$ (data not presented here). Interestingly, available data on the elastic response of the glycocalyx are terse, but there is rather a consensus that the elastic constant lies in the range 2-20 Pa [6,7]. Typical shear stresses being of order $\eta\gamma \simeq 2N.m^{-2}$ in the post-capillary veinules [7], we obtain a value of order 0.1-1 for C_w, which is in good agreement with the value we found (see figure 2, right). The natural elastic modulus of the glycocalyx layer thus seems to induce a maximal repulsion of the RBC from the veinule walls, which could prevent undesirable sedimentation for exemple.

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