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*Summary*The behavior of a driven granular gas in a container consisting of M connected compartments is studied employing a microscopic kinetic model. After obtaining the governing equations for the occupation numbers and the granular temperatures of each compartment we consider the various dynamical regimes. The system displays interesting analogies with the ordering processes of phase separating mixtures quenched below the their critical point. In particular, we show that below a certain value of the driving intensity the populations of the various compartments become unequal and the system clusterizes. Such a phenomenon is not instantaneous, but is characterized by a time scale, τ, which follows a Vogel-Vulcher exponential behavior. On the other hand, the reverse phenomenon which involves the "evaporation" of a cluster due to the driving force is also characterized by a second time scale which diverges at the limit of stability of the cluster.

We shall consider a granular gas [1-2] subjected to a vigorous shaking and initially equi-partitioned into several identical compartments and show that it presents a phenomenology resembling that of spinodal decomposition. At some instant $t = 0$ the shaking intensity is decreased and the system evolves towards a new statistically steady state. The system has two possibilities: one is to persist in the homogeneous state, the second to clusterize [3]. The crossover between the two regimes occurs at a particular value of the driving through the amplification of long-wavelength fluctuations. However, the analogy with spinodal decomposition is not complete, since in the late stage the order parameter does not saturate and the behavior of the granular system shows substantial differences with respect to familiar ordering systems.

We propose a simple extension of a model employed elsewhere [4-5] in order to study the steady state properties of a vibro-fluidized granular gas to the case where the total volume available to the particles is divided into a series of identical compartments and the grains can move from one to the other by jumping over a vertical wall. We consider an assembly of $\mathcal{N}$ inelastic hard-spheres moving in a d-dimensional domain partitioned into M identical regions of volume, V separated by vertical obstacles. Each compartment contains N_i particles and $\sum_{i=1}^{M} N_i = \mathcal{N}$ and belongs to a one dimensional array. The dynamics of the k-th particle between two successive collisions is based on a Langevin type equation of motion for each grain with a fluctuating random force accounting for the action of the external driving force. In addition, the particles contained in a compartment can migrate into neighbor compartment with a probability per unit time, τ_s^{-1}, provided their kinetic energy exceeds the fixed threshold T_s, which is related to the gravitational energy necessary to overcome the vertical barrier separating two neighboring compartments.

The velocity distributions function for the particles belonging to compartment i is $f_i(\mathbf{v}, t)$ and can be obtained by means of a Boltzmann-like kinetic approach. Its evolution is given by

$$\partial_t f_i(v_1,t) = I(f_i,f_i) + \frac{T_b}{\tau_b}\left(\frac{\partial}{\partial \mathbf{v}_1}\right)^2 f_i(\mathbf{v}_1,t) + \frac{1}{\tau_b}\frac{\partial}{\partial \mathbf{v}_1}(\mathbf{v}_1 f_i(\mathbf{v}_1,t)) -$$

$$\frac{1}{\tau_s}\theta(|\mathbf{v}_1| - u_s)[2f_i(\mathbf{v}_1,t) - f_{i+1}(\mathbf{v}_1,t) - f_{i-1}(\mathbf{v}_1,t)] \tag{1}$$

The problem can be reduced to a simple set of governing equations for the occupation numbers and the granular temperatures of the various compartments.

$$\frac{\mathrm{d}N_i(t)}{\mathrm{d}t} = \frac{1}{\tau_s}[N_{i+1}e^{-T_s/T_{i+1}} + N_{i-1}e^{-T_s/T_{i-1}} - 2N_i e^{-T_s/T_i}] \tag{2}$$

$$N_i\frac{\mathrm{d}T_i(t)}{\mathrm{d}t} = \frac{1}{\tau_s}[2(N_{i+1}T_{i+1}e^{-T_s/T_{i+1}} + N_{i-1}T_{i-1}e^{-T_s/T_{i-1}} - 2N_iT_ie^{-T_s/T_i})$$

$$+(N_{i+1}e^{-T_s/T_{i+1}} + N_{i-1}e^{-T_s/T_{i-1}} - 2N_ie^{-T_s/T_i})(2T_s - T_i)] - 2\gamma\omega_i N_i T_i + \frac{2}{\tau_b}N_i(T_b - T_i) \tag{3}$$

where the local dissipation rate is

$$\gamma\omega_i = \sigma(1-\alpha^2)\frac{N_i}{2V}\sqrt{\frac{T_i}{m}} \tag{4}$$

In the case of M identical compartments with cyclic boundary conditions, the choice $N_i = N^*$ and $T_i = T^*$, where T^* and N^*, related by the relation

$$T^*[1 + \tau_b\sigma(1-\alpha^2)\frac{N^*}{2V}\sqrt{\frac{T^*}{m}}] = T_b,$$

represents a uniform solution of eqs. (2)-(3), for all values of the control parameters.

On the other hand, it turns out that such uniform solution is stable only at high temperature, where a diffusive mechanism tends to restore any small perturbation with respect to the homogeneous state. On the contrary, the uniform state below a certain temperature turns out to be unstable with respect to spontaneous fluctuations, due to the clusterization mechanism induced by inelasticity.

N, for large values of T_b, such that $T_g > T_c$, the initial perturbation is re-adsorbed diffusively, while at small values of T_b ($T_g < T_c$) the perturbation is exponentially amplified. In the latter case, the collisional cooling determines a decrease of the local temperature in correspondence of the regions more populated and clusterization begins. Some compartments, randomly selected by the dynamics, act as germs for the nucleation process. After the initial regime few compartments grow at the expense of the remaining which become empty. The distance between highly populated compartments increases, since they compete for particles.
A quantitative measure of the clusterization phenomenon is represented by the following statistical indicator

$$h = -\sum_{i}^{M} \frac{N_i}{N} \ln\left(\frac{N_i}{N}\right) \tag{5}$$

The "entropy" h is non negative, vanishes when all particles are confined in a single compartment and takes on its maximum value, $\ln(M)$, when all compartments are identically populated. Thus $f = \exp(h)$ represents a measure of the number of occupied compartments.
Above T_c the indicator f relaxes toward M, whereas in the low temperature region due to clusterization f decreases toward a plateau value $P < M$. Interestingly, such a relaxation time τ increases as the system approaches the temperature T_c from below. The temperature dependence of τ close to T_c, is consistent with the Vogel-Fulcher law

$$\tau_{vf} = A \exp[\Delta/(T - T_o)]. \tag{6}$$

We turn, now, attention to a different process obtained by considering the evolution of an initial configuration, in which all the particles are located inside a single compartment at $t = 0$. The temperature dependence of the average time τ necessary to wash-out the initial single cluster configuration, now, diverges as $\tau = C/(T_b - T_p)^{3/2}$ at the crossover temperature T_p. In particular the occupation number versus time shows a plateau when $T_b \to T_p$ The smaller the temperature deviation from the limit of stability T_p the longer the plateau.
To summarize, we introduced a model for compartmentalized driven granular gases and studied it using the methods of kinetic theory. We have found a rather rich "phase" behavior and the emergence of new qualitative properties as the number of particles becomes sufficiently large. We have pointed out that the system undergoes a long-wave length instability and orders in a fashion similar to the process which occurs during the spinodal decomposition in fluid mixtures. However, the late stage of the process is radically different, because the granular gas does not possess a surface tension mechanism which restores homogeneity. Thus the usual competition between bulk and surface free energy cost which determines the growth of larger and larger domains is not at work here.

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