GRAVITY WAVES IN A FLUIDIZED SUSPENSION

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Summary A suspension of solid particles fluidized in a liquid is classically described as an effective fluid. With this description, a solid liquid fluidized bed consists of a light fluid (suspending fluid) over a denser and more viscous one (suspension). In the case of two real fluids, the waves generated at their interface are damped. We generate such waves in a Hele-shaw cell, and study the attenuation rate and the phase velocity of the waves as functions of concentration of solid phase of the suspension, and for different viscosities of the liquid. The results are compared to theoretical predictions obtained with the use of the Navier-Stokes-Darcy equation. In particular, an estimation of the effective viscosity of the suspension is obtained from the attenuation rate measurements.

Introduction: Macroscopic suspension are encountered in many natural phenomena (e.g. sedimentation or mixing in estuary) or industrial (food, pharmaceutical, petroleum ...). In most cases suspensions are described as effective fluids of effective density, viscosity and diffusion coefficient depending of the solid phase volume fraction. With this description the interface of a sediment front should behave like the interface between two immiscible fluids but with zero surface tension. Previous author have already put in light such behavior of the interface. Once can cite the work of T. Loimer & U. Schaflinger [1] who studied the effect of shear instability in stratified suspension or more recently the work of C. Völtz *et al.* [2] who showed that suspension interface can undergo Rayleigh-Taylor instability. Fluidized beds is a very powerful tool to study the behavior of "surface" of a suspension [3]: an upward liquid flow compensates the sedimentation velocity, and provide a stationary front. Moreover, the volume fraction of the suspension is easily controlled by the fluid flow rate: high flow rate leads to low volume fraction.

Experimental set-up: The experimental set-up is presented on figure 1. The cell consists of two parallel glass plates, 50×50 cm long and 5 mm thick, close to each other (Hele-Shaw cell). The fluid (water or water-glycerol mixture) is injected at the bottom of the cell through a porous media that performes a uniform velocity at the inlet. Two sets of spherical glass beads have been used, of diameter $d \in [100, 125]$ μm and $d \in [200, 250]$ μm respectively. In order to keep the Reynolds number $\left(\mathrm{Re_p} = \frac{\mathrm{U_{fluid}\, d}}{\nu_{\mathrm{fluid}}}\right)$ small, the smallest volume fraction which has been studied here is $\phi \approx 0.26$. The experiment is illuminated from behind and images are taken with a Firewire CCD video camera located on meter from the experiment and perpendicular to the cell. Images are acquired with the Image J[1] Plugin Fwcamakiz[2] and analyzed with Image J.

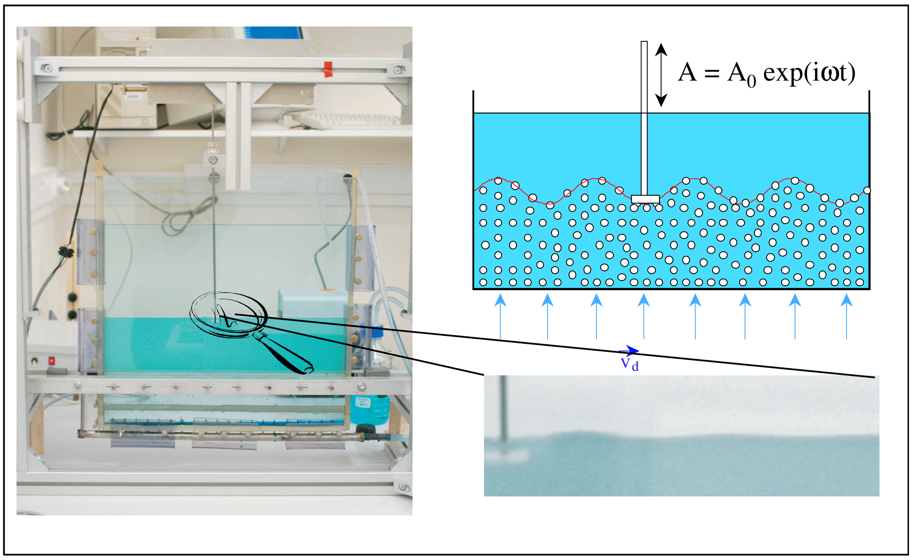

Figure 1. Experimental set-up

The perturbation is created by a small cylinder (1.5 cm long and 4 mm in diameter) located at the interface. The perturbation of the interface is realized via a camshaft driven in rotation by a stepping motor. The so generated waves have a frequency fixed by the rotation rate of the motor $(f \in [0.5, 5]$ Hz$)$, and an amplitude which depends on the camshaft. In

[1] http://rsb.info.nih.gov/ij
[2] http://www.pmmh.espci.fr/ daerr/progs.html#FWCamAkiz

order to change the amplitude of the perturbation two camshafts have been used, generating amplitudes of 1 cm and 2 cm respectively (however, no difference on the wave number and the damping rate has been observed). For sufficiently small perturbation, the height of the interface reads: $h(x,t) = h_0 \exp\left(i\left(kx - \omega t\right)\right)$, where ω is the pulsation of the wave and $k = k_r + i\,k_i$ the wave number; then wave are damped for positive k_i.

Results: As on can seen on figure 1 (bottom right) the system generates waves of small amplitude. We did observe waves only in a small range of frequency ($f \in [1.25, 3.5]$ Hz). We have perform experiments for different solid phase volume fraction ϕ and pulsation and damping rate are presented on figure 2. As the motor of the waves is the gravity, with two fluids the wave length is controlled by the square of the pulsation and the density difference of the two fluids. As one can see on figure 2 the interfacial wave between a suspension and a real fluid keep this property. In the same way, the damping rate (k_i) should be related to the viscosity of the fluids, but due to Hele-Shaw cell, the link between them is not straightforward.
Indeed, at low Reynolds number, flow in a Hele-Shaw cell is described with the Stokes-Darcy equation [4]:

$$\rho \frac{\partial \vec{v}}{\partial t} = -\vec{\nabla} P + \mu \Delta \vec{v} + g\delta\rho - \frac{\mu}{K_{\text{eff}}} \vec{v}$$

with an effective permeability K_{eff} which reads :

$$K_{\text{eff}} = e^2 \left(\frac{\frac{2}{\xi e} \tan\left(\frac{\xi e}{2}\right) - 1}{2\xi e \tan\left(\frac{\xi e}{2}\right)} \right)$$

where $\xi = (1+i)\sqrt{\frac{\omega}{\nu}}$, ω is the pulsation of the wave, ν the viscosity of the fluid and e the thickness of the cell. Then in our experiment the effective permeability is different in the suspension and in the suspending fluid. Moreover as the effective permeability depends of the pulsation ω, we cannot invert the problem to obtain a measure of the effective viscosity. However, thanks to Mathematica [3] we can numerically solve the equation, and then test different viscosity model. Figure 2 show the damping rate k_i as the function of the wave number k_r. The points correspond to experimental data while the continuous line is the result of the two fluids model with an effective viscosity formulae due, to Ball & Richmond [5]: $\eta_{\text{s}} = \eta_{\text{fluid}} \left(1 - \frac{\phi}{\phi_{\text{m}}}\right)^{3/2}$. The agreement between experimental data and fluids model is very good, showing that the effective fluids model is relevant to describe interfaces.

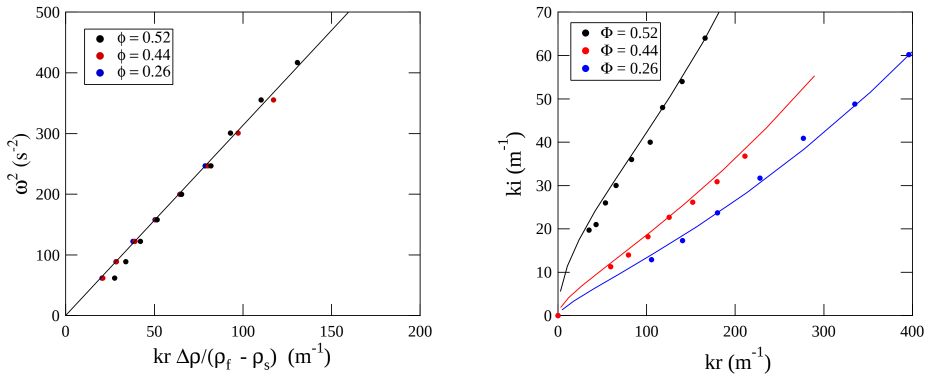

Figure 2. Evolution of the square of the wave pulsation (ω^2) and of the damping rate (k_i) as a function of the wave number (k_r) at the pseudo-interface between the fluidized suspension and the clear fluid. (●) experimental , (—) theoretical.

References

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[3]http://www.wolfram.com