RELAXATION TIME FOR SEDIMENTING SPHERES OF A SUSPENSION WITH PERIODIC BOUNDARY CONDITIONS

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Summary We numerically simulate sedimentation of a suspension with periodic boundary conditions. We consider mono-disperse non-Brownian spheres in low-Reynolds-number fluid flow. We analyze how fast a steady state is reached and how the relaxation time depends on initial positions of the spheres, for a fixed low volume fraction.

INTRODUCTION

Sedimentation of suspensions made of spherical mono-disperse non-Brownian particles has been recently extensively investigated experimentally, theoretically and numerically [1]. The main interest has been to investigate the nature of the steady state and to understand how does it differ from the equilibrium one, characterized by random positions. Explanation of the observed particle velocity fluctuations and theoretical derivation of the relevant pair probability distribution at the steady state are still the open problems.

The question how fast and in what way sedimenting suspensions become stationary is even more challenging. Some authors claim that in some cases a steady state is not reached at all [2]. Experiments of Guazzelli et al. [3, 4] indicate that there exist two characteristic time scales in evolution of a sedimenting suspension. First, after several Stokes times only, a big swirl is formed in the container. Later, after a few hundreds of Stokes times, the system becomes stationary. The relaxation time to a steady state measured in Refs. [3, 4] is of the same order as the corresponding value calculated by Ladd in his numerical simulations [5, 6]. The problem how the relaxation time depends on the size of the system has been raised and investigated [2, 6].

The results obtained for single groups of several spheres only indicate that there may be important to consider also another problem. As it has been shown in Ref. [7], dynamics of three spheres settling under gravity through the Stokes fluid, and in particular their interaction time, are very sensitive to initial conditions. Therefore in this work we investigate if a similar property holds for a suspension. So as we analyze how does the relaxation time of a sedimenting suspension depend on its initial state.

To address this question, in this paper we investigate numerically evolution of a simple model of a suspension sedimenting in cubic periodic boundary conditions. The advantage of numerical approach is that it is possible to specify arbitrary initial conditions, even very far from equilibrium. Therefore we do not restrict to a random initial state with the uniform distribution of positions of the sphere centers. On the contrary, we consider three essentially different classes of initial positions of the sphere centers,

(R) chosen at random,

(L) slightly and randomly perturbed from cubic lattice, and

(P) randomly grouped into close pairs, which are in turn randomly distributed within the cube.

THEORETICAL BACKGROUND

The fluid is described by the incompressible Stokes equations with periodic boundary conditions and the stick boundary conditions at the surfaces of N spheres immersed in the periodic cube. The Green tensor for such a system has been specified by Hasimoto [8] and expressed in a form suitable for fast numerical computation by Cichocki and Felderhof [9]. Their results are implemented in the numerical codes used in this paper. For given positions of N spheres, the mobility coefficients are evaluated by the multipole method [10], with the lubrication correction [11], which takes care of interactions between close spheres. The algorithm and the numerical code described and applied e.g. in Ref. [12] for infinite fluids, here are modified and used to account for periodic systems. Equations of motion for N spheres are integrated with the fourth-order Runge-Kutta algorithm. In this way the particle positions and velocities are determined as functions of time.

NUMERICAL SIMULATIONS

In most of the simulations, we consider the small volume fraction ϕ=0.003. This value corresponds to the system investigated experimentally in Refs. [3, 4]. Because of practical reasons, we restrict to small numbers of particles in the periodic cube, mainly N = 8. However, we trace spheres rather than point particles. To get accurate trajectories, we truncate the multipole expansion at the multipole number $L = 3$ (with L defined in Ref. [10]).

The quantities of interest are the mean sedimentation velocity and the mean horizontal and vertical velocity fluctuations, traced as functions of time for thousands of Stokes times. The averaging is performed over all the particles in the periodic cell and over a number of random initial configurations, separately within each of three classes listed above, with a fixed amplitude of perturbation from cubic lattice, for class (L), and a fixed maximum distance between the sphere centers within each of close pairs, for class (P).

It is shown that the relaxation time to a steady state is sensitive to initial conditions. It is analyzed how does it depend on the amplitude of perturbation from cubic lattice and on the upper bound on the distance between the sphere centers within each of close pairs.

References

[1] Ramaswamy S.: Issues in the statistical mechanics of steady sedimentation. *Adv. Phys.* **50**:297–341, 2001.

[2] Tee S.Y., Mucha P.J., Cipelletti L., Manley S., Brenner M.P., Segre P.N., Weitz D.A.: Nonuniversal velocity fluctuations of sedimenting particles. *Phys. Rev. Lett.* **89**:054501-1–4, 2002.

[3] Guazzelli E.: Evolution of particle-velocity correlations in sedimentation. *Phys. Fluids* **13**:1537–1540, 2001.

[4] Bergougnoux L., Ghicini S., Guazzelli E., Hinch J.: Spreading fronts and fluctuations in sedimentation. *Phys. Fluids* **15**:1875-1887, 2003.

[5] Ladd A.J.C.: Dynamical simulations of sedimenting spheres. *Phys. Fluids A* **5**:299–310,1993.

[6] Ladd A.J.C.: Effects of container walls on the velocity fluctuations of sedimenting spheres. *Phys. Rev. Lett.* **88**:048301-1–4, 2002.

[7] Janosi I.M., Tel T., Wolf D.E., Galas J.A.C.: Chaotic particle dynamics in viscous flows: The three-particle Stokeslet problem. *Phys. Rev. E* **56**:2858–2868, 1997.

[8] Hasimoto H.: On the periodic fundamental solutions of the Stokes equations and their application to viscous flow past a cubic array of spheres. *J. Fluid Mech.* **5**:317-328, 1959.

[9] Cichocki B., Felderhof B.U.: Periodic fundamental solution of the linear Navier-Stokes equations. *Physica A* **159**:19–27, 1989.

[10] Cichocki B., Felderhof B.U., Hinsen K., Wajnryb E., Bławzdziewicz J.: Friction and mobility of many spheres in Stokes flow. *J. Chem. Phys.* **100**:3780-3790, 1994.

[11] Brady J.F., Bossis G., Stokesian dynamics. *Ann. Rev. Fluid Mech.* **20**:111-157, 1988.

[12] Cichocki B., Ekiel-Jeżewska M.L., Szymczak P., Wajnryb E.: Three-particle contribution to sedimentation and collective diffusion in hard-sphere suspensions. *J.Chem. Phys.* **117**:1231–1241, 2002.