THE SLOPPED FALLING FILMS WITH SURFACTANTS: INSTABILITY AND NONLINEAR WAVES

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Summary The film flow of a weak solution of a soluble volatile surfactant in liquid is considered. Diffusion of the surfactant to the film surface from the bulk solute, its evaporation to the gas phase and the adsorption-desorption processes in the near-surface layer are taken into account. The full Navier-Stokes formulation is reduced to the system of nonlinear evolutionary equations. A model for the dynamic surface tension is appropriate for the transition of the freshly formed surface to the "old" surface to be investigated. A steady state solution for film flow along a slopped wall and instability of the flow are considered for the simultaneous action of the body forces, capillary pressure and Marangoni stresses. The nonlinear effects for the hydrodynamic and diffusion instability modes for non–equilibrium adsorption kinetics in the sorbed sublayer are investigated.

INTRODUCTION

The flow of a falling film of an aqueous surfactant solution along a slopped wall with surfactant adsorption-desorption at its open surface inducing Marangoni effect is investigated. The diffusion of surfactant to the film surface from the bulk and desorption of surfactant to the gas phase are taken into account. The bulk concentration of surfactant c, bulk surfactant concentration $\bar{c}$ in the fluid sublayer, surface excess concentration Γ in the adsorbed layer on the interface are introduced. The surface tension coefficient σ is supposed to be a linear function of the both values $\bar{c}, \Gamma$. It gives an opportunity of varying conditions from "new" to "old" surface along the liquid flow to be taken into account. In [1, 2] for vertical slope the Navier–Stokes and Fick equations together with appropriate boundary conditions are reduced to a system of simpler hence analytically and numerically more tractable nonlinear evolutionary equations. The linear stability analysis for various values of the significant dimensionless parameters has revealed a very rich picture of instability modes. In addition to the earlier known hydrodynamic mode there are up to four new Marangoni–driven diffusion modes. One diffusion mode could be identified as a monotonic mode hence leading to a patterned film surface. All other modes are oscillatory ones. Resonance of modes is also predicted for suitable combinations of the parameters. The mode observed depends upon the particular choice of the adsorption–desorption kinetics and the surface tension state equation at the open surface of the film.

MATHEMATICAL MODEL

We solve the closed basic model system [1, 2] for $h, \overline{u}, \overline{c}, \overline{\Gamma}, h_1$

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0$$

$$\frac{\partial q}{\partial t} + \frac{\partial Q}{\partial x} = \frac{1}{5\delta}\left[h\frac{\partial^3 h}{\partial x^3} + h - \frac{2}{3h}\left(\overline{u} + \mathrm{M}h\overline{c}_x\right)\right] - \chi h\frac{\partial h}{\partial x}$$

$$\frac{\partial \varphi}{\partial t} + \frac{\partial}{\partial x}\left[\left(A\overline{u} + B\mathrm{M}h\overline{c}_x\right)\varphi\right] = 2\frac{\overline{c}}{\Delta}$$

$$n_*\mathrm{G}\left[\frac{\partial \Gamma}{\partial t} + \frac{\partial}{\partial x}\left(\overline{u}\Gamma\right) + \frac{\partial \overline{u}}{\partial x} - n_*^2\mathrm{Di}\frac{\partial^2 \Gamma}{\partial x^2}\right] + \mathrm{Bi}\left(1 + \overline{c}\right) = -2\frac{\overline{c}}{\varepsilon\Delta}$$

$$\pi_1\left(1 + \overline{c}\right) - \pi_2\left(1 + \Gamma\right) = -2\frac{\overline{c}}{\varepsilon\Delta},$$

where

$$q = \int_0^h u dy = \frac{2}{3}\overline{u}h + \frac{1}{6}\mathrm{M}h^2\overline{c}_x$$

$$Q = \int_0^h u^2 dy = \frac{8}{15}\overline{u}^2 h + \frac{7}{30}\mathrm{M}h^2\overline{u}\overline{c}_x + \frac{1}{30}h(\mathrm{M}h\overline{c}_x)^2$$

$$\varphi = \int_0^\Delta c d\zeta = \frac{1}{3}\overline{c}\Delta$$

with

$$A = 1 - \frac{1}{10}\frac{h_1^2}{h^2}, \quad B = \frac{1}{4}\frac{h_1}{h} - \frac{1}{10}\frac{h_1^2}{h^2}, \quad h_1 = \varepsilon\Delta\,.$$

Here h, h_1 are the thickness of the film and of the diffusion boundary layer, $\bar{u}$ is the velocity of liquid at the open surface. The solution to the basic model system depend upon ten non–dimensional parameters which reflect the influence of the main processes included in the mathematical model of film flow

$$\mathrm{Ma},\ \delta,\ \mathrm{G},\ \mathrm{Bi},\ \mathrm{Di},\ \varepsilon,\ \pi_1,\ \ \pi_2,\ \chi,\ \ n_*.$$

A wave number α is to be included to the list of free parameters for periodical solutions in coordinate x along the flow. The main parameter connecting the hydrodynamic and diffusion parts of the film flow problem with surfactant is the Marangoni number Ma. Both cases, positive ($\mathrm{Ma} > 0$) and negative ($\mathrm{Ma} < 0$) Marangoni numbers have been considered. For $\mathrm{Ma} = 0$ we obtain the system of paper [4] for pure liquid film with free surface. For $\mathrm{Ma} \neq 0$ the first two equations coincide with the systems obtained in [5, 6] for film flows under the action of tangential force τ, where $\tau = \mathrm{M}h\bar{c}_x$, $\mathrm{M} = n_*\mathrm{Ma}$. The most significant hydrodynamic parameters are δ and n_*. Their corresponding values determine the mean film thickness l, mean velocity U_* and flow rate lU_*. The diffusion parameters, $\bar{c}_0$ and ε, determine the local thickness of the diffusion boundary layer, h_1. Three quantities, π_1, π_2, and Di, characterize the mass transfer of surfactant by the adsorption-desorption and the intensity of dissipation by the surface diffusion. The intensity of the surfactant desorption to the gas phase is determined by the parameter Bi. The remaining parameter G gives an indication of the typical value of surface excess concentration Γ_* relative to c_*. It is useful the parameter $\mathrm{T} = n_*\mathrm{G}/\pi_2$ to be introduced. For $\mathrm{T} \to 0$ the case of diffusion controlled adsorption-desorption kinetics is obtained: $\Gamma = \bar{c}$. This corresponds to a fast desorption process leading to local kinetic equilibrium. In the opposite limiting situation, $\mathrm{T} \to \infty$, we obtain the case of the kinetically frozen desorption. One of the ten parameters of the problem that has not been considered in [1–3] is the angle of the wall inclination χ.

The system of evolutionary equations by the numerical methods is solved. We apply the Fourier series for homogeneous coordinate x together with numerical integration for Cauchy problem in time t. Examples of the linear instability modes in [1–3] are given, while in [5,6] the non-linear evolution of waves under the action of tangential forces on the film surface are demonstrated. The findings of the papers [1–3] put forth new problems concerning a variety of instability modes in films with surfactants. The nonlinear development of the hydrodynamic (transverse) and the Marangoni–driven (longitudinal) waves of the finite amplitude is the first one among those. The asymptotic behavior of the Marangoni–driven waves as the wave number grows is the second problem to be investigated in more details. The parametric experimental investigations of the linear and nonlinear instability waves could be correctly organized. All those questions are investigated and presented in the talk.

The work was financially supported by RFBR (Grants 00-01-00645, 03-01-00042)

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