

## LIQUID FILM FLOW IN CONICAL CAPILLARY

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The phenomenon of two-side filling with some liquids of conical capillaries was established in 1989 [1]. During the last years comprehensive experimental data concerning this phenomenon were obtained. But till the present time there is a lack of its theoretical description. At first we tried to explain the phenomenon on the base of the existence of the difference between the saturated vapour pressures above two menisci in dead-end capillary. It results in the evaporation of a liquid from the meniscus of smaller curvature ("classical" capillary imbibition) and the condensation of its vapour upon the meniscus of larger curvature originally existed due to capillary condensation. We worked out the mathematical description of both gas-vapour diffusion and evaporation-condensation processes in cone's channel. This theoretical model was presented at 19<sup>th</sup> ICTAM in Kyoto [2]. But the calculated values for inner column's length are 1-2 orders of magnitude smaller than the experimental ones.

Now we present the model, which corresponds to liquid film flow that occurs in the adsorption liquid layer along the channel's wall. This flow might be directed only towards the cone's top because of the decrease of the pressure in a liquid film with the increase of meniscus curvature. Theoretical model of the film flow in conical capillary is based on the concept of disjoining pressure  $\Pi$  in thin liquid film [3]. This film is stabilized by the presence of disjoining pressure. The moving force of a flow is the difference of the pressures in a liquid film near two menisci of different curvatures.

We use the approximation for the isotherm of disjoining pressure  $\Pi(h) = A/h^3$ , where  $A$  is the constant, and solve Navier-Stokes equation for film flow with the gradient of corresponding pressure as the moving force. After some transformations we obtain differential equation for a film thickness  $h$ :

$$-\left(\frac{3\mu Q}{2\pi} \frac{1}{h^3(r)} + \frac{\sigma}{r}\right) \frac{h^4(r)}{3A\tau} = \frac{dh(r)}{dr},$$

which should be solved in the range of radii  $R_2 \leq r \leq R_1$  under the boundary conditions connecting the pressure in the film on the boundaries of both menisci with the pressures in a bulk phase of corresponding liquid column:

$$p_i(R_1) = p_g - \frac{2\sigma}{\tau R_1}, \quad p_i(R_2) = p_g - \frac{2\sigma}{\tau R_2}.$$

Here  $r$  – radial axis in spherical coordinate frame;  $p_g$  – pressure of a gas above the film;  $R_1$  and  $R_2$  – menisci coordinates;  $\tau = r_0/R_c$ ;  $R_c$  – capillary length;  $Q$  – liquid's flux through a whole film cross section, which is independent on  $r$ .

After some transformations we derive the formula for the time dependence for the volume of top's liquid column

$$t = \frac{\tau\mu}{4A} R_1^3,$$

where  $\mu$  - viscosity coefficient.

Our theoretical results obtained in the paper are in satisfactory agreement with experimental data [4] and confirm the idea that liquid film flow has decisive importance in the studied phenomenon.

### REFERENCES

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