LINEAR STABILITY OF THE INCOMPRESSIBLE SWEPT HIEMENZ FLOW : A $2D$ MODEL

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Summary A $2D$ approach of the linear stability theory confirms that the most amplified mode for the swept Hiemenz flow is the one given by the Görtler-Hämmerlin model. A least amplified $2D$ mode, with a close frequency to the latest emerged, that could play a role in the transition process. Though, others $2D$ modes, evoked in previous works, are not observed.

GENERAL CONTEXT

The considered flow is the incompressible one developing around the attachment-line of a swept wing, see Fig.1. The $3D$ basic flow is steady and depends on both coordinates x and y ; it is usually called the swept Hiemenz flow. There is a particular perturbation which has the same similarity form as the one of the mean flow, the so-called Görtler-Hämmerlin (GH) mode, which has been extensively studied in the past.

The main purpose of the present work is to develop a general two-dimensional approach for the linear stability theory for any basic flow depending on x and y. The objective is thus to determine all possible $2D$ eigenmodes. The GH mode is expected to be among them.

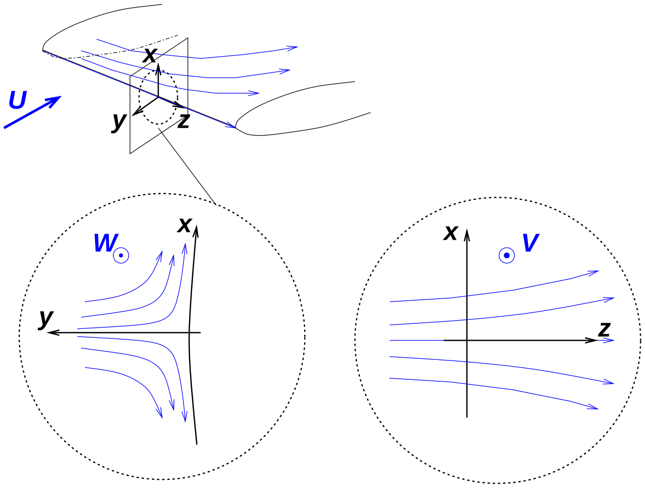

Figure 1. Swept Hiemenz flow : geometry and axis definition

Pressure and velocity fluctuations, superimposed to this basic state, are chosen periodic in time and in spanwise direction : $q(x,y) = \hat{q}(x,y) exp i\beta(z-ct)$. In the temporal study, β is real and represents the transverse wave length, and c is the complex phase velocity, its real part represents the propagating speed and its imaginary part is related to the amplification rate. After the linearization of the Navier-Stokes equations we obtain a set of partial differential equations. One can either use the primitive variables or combine the equations in order to obtain a reduced system for the u and v fluctuating velocity components. The physical boundary conditions express the no-slip condition at the wall and that the perturbation vanishes far from the wall.

NUMERICAL ASPECTS

Chebyshev spectral collocation method is used in both directions. The grid is built with Gauss-Lobatto nodes. They are then stretched in the wall normal direction in order to refine point distribution in the boundary layer. The unknown functions are approximated with an interpolation of discrete values at the collocation nodes by Lagrange polynomials.

In addition to previous boundary conditions, outflow conditions are needed. Different ones have been tested, such as a linear extrapolation scheme. As pressure and velocity components are defined on the same nodes, the system is undetermined and artificial compatibility conditions are then added to the set of discrete equations. Finally, the discretized system is represented by a general eigenvalue problem $Ax = cBx$. Parity properties are also exploited in order to reduce the size of A and B and hence improve computational precision. In any case, this size remains huge in comparison with the $1D$ stability problem. Thus, the eigenvalue c and its associated eigenvector are determined using the Arnoldi algorithm. The computation cost becomes indeed rapidly prohibitive if we use a QZ algorithm. The Arnoldi method builds a Krylov subspace of orthogonal vectors and its associated Hessenberg matrix ; this provides a small part of the original spectrum, but with a very acceptable cost.

$2D$ RESULTS

We focus on the least stable part of the spectrum, $c_i > 0$ gives amplified modes, $c_i < 0$ attenuated ones. Two examples are given in Fig. 2, they concern a flow with a Reynolds number $R = 800$ and a transverse wave length $\beta = 0.255$ (reference length and velocity are the ones used in [1] and [2]). In the left-hand side figure, different spatial resolutions for a fixed size of computational domain ($[0, maxX] \times [0, maxY]$) have been tested. Some points are clearly independent of the grid resolution. However, looking at the right-hand side figure, which gives the influence of $maxX$, it can be infered that only the two most amplified modes are independent of the numerical parameters.

These two modes match correctly those given by other $2D$ models [1], [2]. They are represented by circles in the graph. The least stable one is symmetric, called S1, this is actually the Görtler-Hämmerlin mode. The corresponding velocity fluctuation has a longitudinal component that depends linearly on the chord position (see Fig. 3) while the normal and

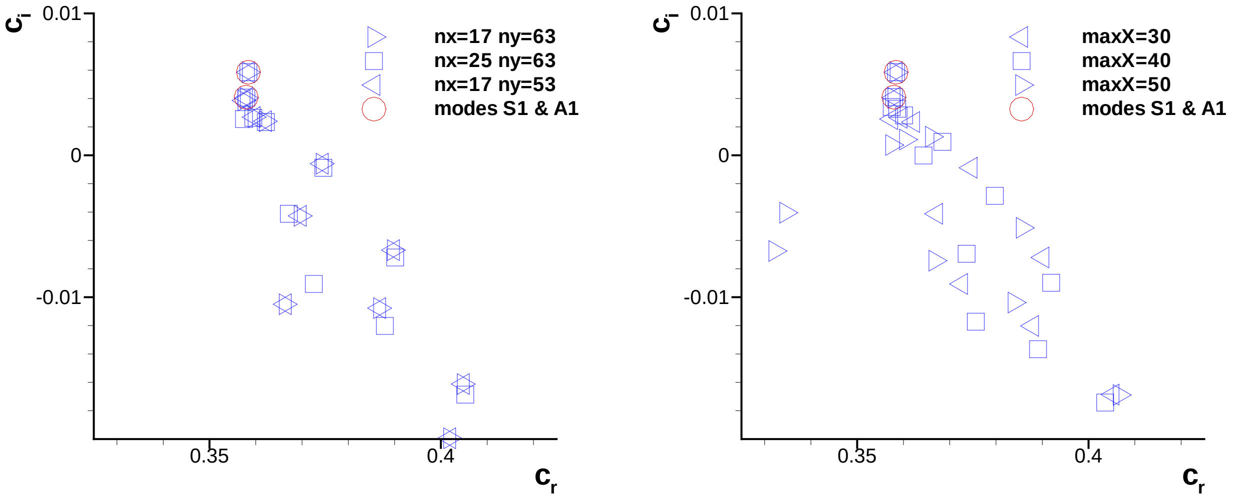

Figure 2. Spectra - $R = 800,\ \beta = 0.255$ - On the left : $maxX = 40$, on the right : $nx = 25, ny = 63$

	c_r	$c_i\ (\times 10^{-2})$		c_r	$c_i\ (\times 10^{-2})$
S1 in [1]	0.35840982	0.58532472	A1 in [1]	0.35791970	0.40988668
nx=17-ny=53-maxX=40	0.35840994	0.58516640	nx=17-ny=53-maxX=40	0.35797297	0.40359194
nx=17-ny=63-maxX=40	0.35840943	0.58512904	nx=17-ny=63-maxX=40	0.35797257	0.40355064
nx=25-ny=63-maxX=40	0.35840943	0.58512904	nx=25-ny=63-maxX=40	0.35796069	0.40655494
nx=25-ny=63-maxX=30	0.35840943	0.58512903	nx=25-ny=63-maxX=30	0.35794459	0.40509869
nx=25-ny=63-maxX=50	0.35840943	0.58512904	nx=25-ny=63-maxX=50	0.35798359	0.40967769

Table 1. Eigenvalues S1 and A1 for diffent configurations

transverse components are independent of it. The second mode is an antisymmetric one, called A1. The normal and transverse velocity components grow linearly with the chord position, and the longitudinal component depends on chord position too, in a kind of quadratic way (see Fig. 3). Table 1 summarizes the two converged eigenvalues.

The A1 mode, which corresponds to a $2D$ mode, seems to grow faster in the chord direction than the most amplified mode S1. It could probably play a role in the transition process, even if it is more attenuated with respect to time. Lin & Malik [1] and Theofilis *et al.* [2], who have also developed a $2D$ formulation of the stability of swept Hiemenz flow, find however a series of least amplified modes, alternately even and odd with frequencies close to the GH mode one. The main difference between all these numerical approaches is the chord domain discretisation and the outflow boundary condition. Different initial systems (primitive variables and reduced system), different boundary conditions have been tested. And in all cases, we only obtain two amplified modes. Consequently, we have some doubts about the real existence of the modes, except S1 and A1, published in [1] and [2].

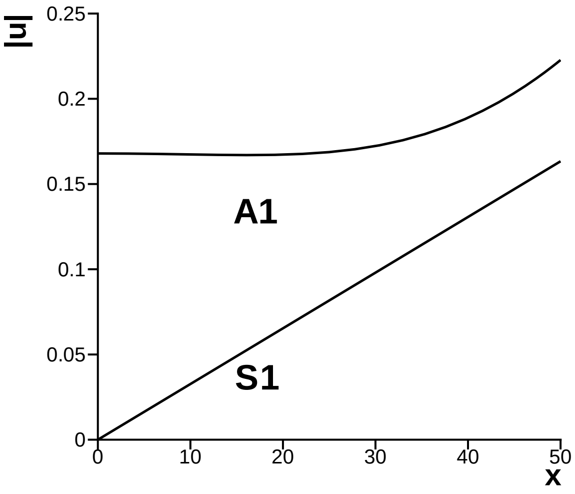

Figure 3. Evolution of the longitudinal velocity component for S1 and A1 modes at a given y close to the third of the boundary layer

References

[1] Lin R.S., Malik M.R.: On the stability of attachment-line boundary layers. Part1. The incompressible swept Hiemenz flow. *J. Fluid Mech* **311**:239–255, 1996.

[2] Theofilis V., Fedorov A., Obrist D., Dallmann U.Ch.: The extended Görtler-Hämmerlin model for linear instability of three-dimensional incompressible swept attachment-line boundary layer flow. *J. Fluid Mech* **487**:271–313, 2003.