SCALING WITH FREESTREAM FLUCTUATIONS IN THE LAMINAR-TURBULENT TRANSITION PROCESS

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Summary It has been known for a century now that transition to turbulence is delayed when background disturbances are decreased. In boundary layers in quiet tunnels, the transition onset Reynolds number and the amplitude of freestream disturbance are related by a power law, with an exponent of -1. Recent experiments in pipe flows too show a power-law relationship between these two quantities, with the same exponent. We investigate here the relationship between background disturbance amplitude and the critical Reynolds number for a precursor to transition onset, namely, secondary disturbance growth in a channel. For each primary mode, the Reynolds number for a given secondary growth rate is found to obey a power-law with a frequency and growth-dependent exponent. The exponents are negative with magnitudes less than 1; the envelope of instability has an exponent close to -1. We conlude with the surprising observation that critical Reynolds numbers defined using the *disturbance velocity magnitude* as scale, are constant in all these cases.

INTRODUCTION AND MAIN CONCLUSIONS

It has long been known that a decrease in the background disturbance delays the onset of turbulence. This was shown first by O. Reynolds in 1905 [1] when he showed that transition in a pipe occurred at higher Reynolds numbers as the background noise was reduced, the envelope over which this holds was extended by Nishioka *et al.* [2]. In boundary layers, Schubauer & Skramstad [3] and many others see eg. [4] demonstrated similar behaviour. The same trend holds true for the onset of chaos in nonlinear dynamical systems as well [5]. Here, we discuss the response to external disturbance of two events in the process of transition to turbulence (i) the growth of secondary waves, calculated for the flow through a channel, and (ii) the onset of transition, on the basis of a compilation of available data in boundary layers [6], and pipes [7].

We show a striking similarity between the two: a power-law relationship between onset Reynolds number and the level of background disturbance, with an exponent close to -1, seems to be ubiquitous in the onset of transition, as well as in the *envelope* of secondary instability. This suggests that the onset of secondary growth in channels, as well as transition in boundary layers and pipes occurs at a *constant* Reynolds number based on a magnitude of the relevant disturbance velocity.

SECONDARY DISTURBANCE GROWTH

The secondary instability analysis in a channel is conducted in the standard manner [8, 9]. The external disturbance is taken to be broadband [10], and the amplitude A_p of the dominant primary mode thus bears a direct relationship to the level of external disturbance. A few typical results are presented here, and an analysis of the dependence of the exponents on the various parameters will be presented at the conference. The Reynolds number for a primary mode of wavenumber 1 and a given maximum (over all spanwise wavenumbers) temporal growth rate ω_i of the subharmonic mode is plotted in figure 1 (a) against the primary disturbance amplitude. It is seen that a power-law behaviour is obeyed, but the exponent changes with ω_i. In figure 1 (b), the frequency of the primary disturbance has been kept constant. The plots of Reynolds number versus A_p (not shown) all display a striking power-law behaviour, for about a decade in each case. The exponents have been collected and are shown here as functions of the growth rate ω_i. Figure 2 gives the minimum instability Reynolds number (over all streamwise and spanwise wavenumbers, and all secondary modes between subharmonic and harmonic). The primary disturbance used is the least stable mode at a given Reynolds number. The Reynolds number $R_c = v_p H/\nu$ (v_p being the maiximum of normal component of the primary disturbance velocity, H and ν the channel half-widths and kinematic viscosity of the fluid) is constant at ~ 13.

TURBULENCE TRANSITION ONSET

The relationship between transition Reynolds numbers and levels of disturbance in a boundary layer was obtained [6] from a compilation of available experimental data to be $R_t = 110 + 340/q_t$, where R_t is the Reynolds number of transition onset, and q_t is an equivalent disturbance in the tunnel. At low levels of background disturbance, R_t

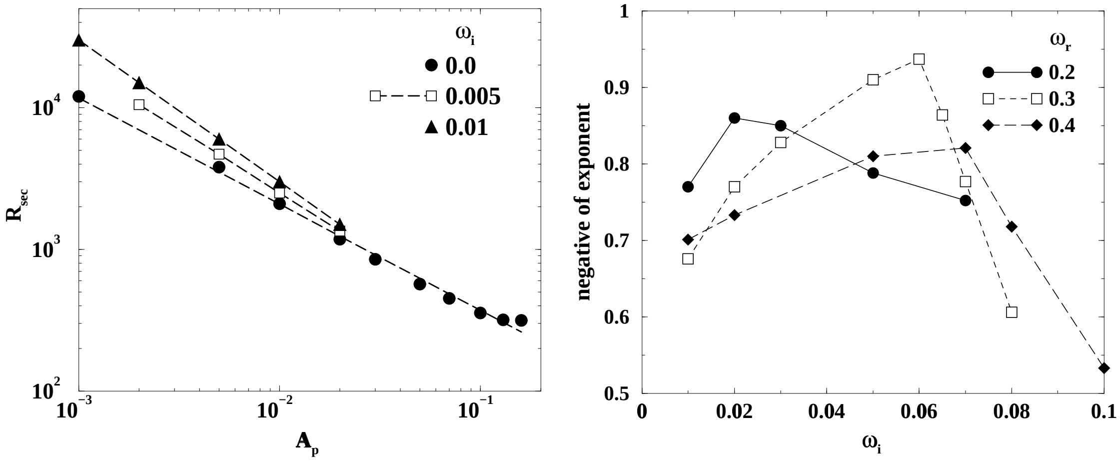

Figure 1. (a) Minimum Reynolds number at which a subharmonic secondary mode grows at a given rate ω_i, the slopes for increasing ω_i are -0.75, -0.89 and -1 respectively. (b) Exponents displayed by Reynolds Vs. A_p when the primary frequency is constant.

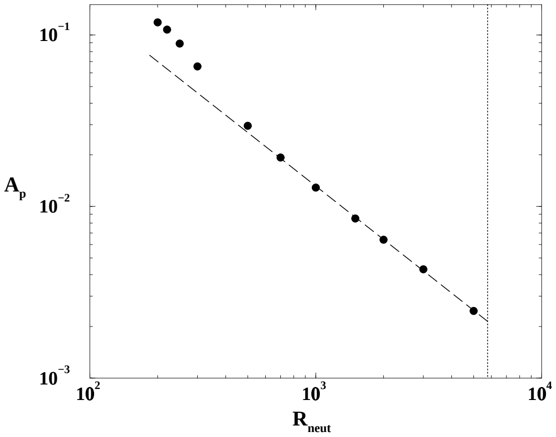

Figure 2. Minimum critical Reynolds number over all secondary modes, as a function of (least stable) primary disturbance amplitude. The slope is -1.03. The vertical line shows the location of linear instability.

varies as q_t^{-1}. Recently Hof, Huel & Mullin [7] found that the transition Reynolds number in a pipe varies as q_p^{-1}, where q_p is the ratio of the fluxes of disturbance and the basic flow.

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