ADVANCES IN MATHEMATICAL MODELING OF HYDRAULIC STIMULATION OF THE HOT DRY ROCK GEOTHERMAL RESERVOIR

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Summary In the present study, network models of "fractal geometry" approximate a 3D structure of fractured rocks. The fracture network models are generated by distributing fractures randomly in space and assuming the fractal equation correlating the number of fractures and fracture lengths. Based on this approach, a mathematical model of hydraulic rock fracturing is proposed. The model incorporates approximations of the fracture mechanical behavior and a very simplified analysis of the operative physical processes.

INTRODUCTION

The numerical model FRACSIM-3D developed by the research group in Tohoku University is proved to be an appropriate approximate model capable to address the problems associated with hydraulic stimulation and can quantitatively predict the 3D reservoir growth behaviour. One of the aspects of the general problem of reservoir modelling is the proper approximation of the fracture distribution within the rock. In fractured rocks, groundwater flow occurs predominantly through the connected network of discrete fractures. A series of geophysical investigations has confirmed that subsurface fracture networks can commonly be described by fractal geometry. The model presented here is based on the relationship between the fracture length and a number of fractures, as suggested in [1]. It incorporates the elements of the approximate model discussed in [2], where the fracture shear displacements and openings, variation of the shape of the stimulating rock volume, pressure compliant fracture apertures are taken into account. The natural rock fracture surfaces, which are proved to be of fractal geometry as well [3], are modelled on the basis of the spectral synthesis method [4], which accounts for the fractal nature of the fracture surfaces. This model is used for numerical simulation of the fracture dilation due to the offset of the fracture surfaces and accounts for the experimentally acquired data of the mating rough fracture surfaces profiles.

MATHEMATICAL MODEL OF THE FRACTURED RESERVOIR STIMULATION

The models of fracture networks are generated by distributing penny-shaped subsurface fractures randomly in space and assuming the fractal correlation $N_r=Cr^{-D}$ that incorporates the fracture length r, number of fractures N_r whose characteristic length is greater than r, fractal dimension D, and fracture density within the rock mass C. Hence, the number of fractures between the specified upper and lower limits is given by $N_{r_{\min}}^{r_{\max}} = C(r_{\min}^{-D} - r_{\max}^{-D})$, where $r_{\min}$ and $r_{\max}$ are the lower and upper fracture radius limits, respectively. Consider some fraction, α, of this total number counting from $r_{\min}$ upward and the corresponding size r_α of the largest object in that fraction, then $N_{r_{\min}}^{r_\alpha} = \alpha C(r_{\min}^{-D} - r_{\max}^{-D}) = C(r_{\min}^{-D} - r_\alpha^{-D})$. The latter yields the fractal fracture length distribution $r_\alpha = [(1-\alpha) r_{\min}^{-D} + \alpha r_{\max}^{-D}]^{-1/D}$, where α is a random parameter in the interval [0,1]. Using this equation, fracture r_α can be generated by simply changing the α value. For any generated fracture, the initial (i.e. undisturbed) fracture aperture, a_0, when evaluated at zero effective stress, is assumed to be proportional to the fracture radius, $a_{i0} = \beta \cdot r_i$, where i=1, 2,…, N_f, and r_i is the radius of each fracture from the whole set of fractures N_f; β is a constant of proportionality, which is chosen to allow the undisturbed fracture network to match (at least approximately) the *in situ* measured permeability. Fracture apertures are affected by the effective normal stress at the fracture surface and by shear displacement that determines the fit of the opposing rough surfaces. Shear stability is expressed by a simple friction law, when slip is taking place if $\tau > (\sigma_n - P)\tan\left(\phi_i + \phi_{dil}^{eff}\right)$, where τ is a shear stress, σ_n is the rock stress normal to the fracture surface, P is a fluid pressure in fracture, and ϕ_{dil}^{eff} is an effective shear dilation angle at a given effective normal stress. The basic friction angle ϕ_i is a material property of the fracture walls. The effective shear dilation angle ϕ_{dil}^{eff} is a property of both the fracture wall asperities and effective normal stress

$$\tan(\phi_{dil}^{eff}) = \tan(\phi_{dil})/[1 + 9(\sigma_n - P)/\sigma_{nref}], \qquad (1)$$

where σ_{nref} is the effective normal stress applied to cause a 90% reduction in the aperture and ϕ_{dil} is the shear dilation angle at very low effective stress. The amount of shear displacement depends on the fracture shear stiffness and on the amount of "excess" shear stress available. Referring to the theory of elasticity, the shear displacement of a fracture U can be expressed as:

$$U = [\tau - (\sigma_n - P)\tan(\phi_i + \phi_{dil}^{eff})]/H_s, \qquad (2)$$

where H_s is a shear stiffness of the fracture. The change in aperture is a product of the displacement and the tangent of the effective shear dilation angle, $a_s = U \tan(\phi_{dil}^{eff})$. An expression for the aperture a of a sheared fracture is following

$$a = (a_0 + U \tan(\phi_{dil})) / [1 + 9(\sigma_n - P)/\sigma_{nref}]. \quad (3)$$

For the typical conditions of the geothermal reservoir exploitation, flow is laminar and the linear Darcy momentum equations can be employed, namely, $u_m = -(K_m / \mu)(\partial P / \partial x_m)$, where m=1, 2, 3 and K_1, K_2, K_3 are the diagonal components of the permeability tensor. Accounting for the mass conservation equation, the Darcy flow model leads to the following equation for pressure distribution

$$\sum_{m=1}^{3} \partial[K_m (\partial P / \partial x_m)] / \partial x_m = 0, \quad (4)$$

The above model contains the unknown value of shear dilation angles ϕ_{dil}, which can be predicted for the specified rock by analyzing the effect of shears displacement of the syntactic fracture on the fracture dilation. This can be made with analytic-experimental method (it couples the analytical algorithm for fracture mathematical modeling and experimental data for the natural fracture topography).

MODELING OF FRACTURE DILATION CAUSED BY SHEAR OFFSET

The algorithm of natural fracture surface simulation involves: (i) measuring the fracture surface topography; (ii) computing the power spectral density for the fracture surface as a function of spatial frequency; (iii) plotting this function in logarithmic coordinates and computing the plot's slope β (1?β?3) and surface fractal dimension $D_S = (5 - \beta)/2$; (iv) applying these parameters to constructing synthetic fractal surfaces that share the same physical properties as the natural fracture surfaces. A detailed description of the spectral synthesis method in application to fractal surface modeling and corresponding computer codes can be found in [3] and [4]. To model the fracture dilation induced by shear offset, two synthetic fractal surfaces that make up the fracture can be assumed to match each other perfectly—the upper surface is been taken to be an exact but inverted duplicate of the lower surface. If these surfaces are laterally offset, they maintain contact at least at one point. If fracture asperities are assumed to be unabrading, then this schematic geometric approach for modeling the shear induced growth of the fracture aperture is quite reasonable [3]. If there is no interpenetration, increasing the lateral displacement of the synthetic surface causes the aperture to increase, which can be readily computed in geometrical terms. Our computations show that fracture dilation is more pronounced for the fractal surfaces with lower fractal dimension (greater β). After the variation of fracture dilation *vs* shear displacement is computed, the shear dilation angle ϕ_{dil} can be obtained (and subsequently substituted into (3)) as the mean angle between the tangent to the shear dilation plot and the horizontal axis. A series of computations was carried out to estimate the effect of the fracture fractal dimension D_s and fracture length on of the shear dilation angle and of the fractal dimension of fracture distribution D and fracture density C on the reservoir volume.

CONCLUSIONS

Conclusions drawn on the basis of a large number of computations are following.
(1) Fracture dilation due to shear offset is more pronounced for the fractal surfaces with lower fractal dimension D_S.
(2) Dispersion of the results computed for the different random distributions is relatively low and ignoring this "noise", the relationship of linear growth of the reservoir with growth of the flow rate can be readily assumed.
(3) Fracture distributions with of lower density lead to larger stimulated volumes for the same flow rates in the stimulating well.
(4) Reduction of the fracture connectivity with respect to fractal dimension D can be approximated by a linear function. For higher fractal dimensions, there are fewer fractures of the maximal radius. This leads to a decrease in the connectivity of the network.
(5) Fractured media with low fractal dimension shows a higher potential for developing relatively big stimulating volumes.

References

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