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A JOULE-THOMSON PROCESS OF A WETTING FLUID NEAR SATURATION

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Summary A fluid with positive Joule-Thomson coefficient flows through a porous membrane. Upstream of the membrane, there is saturated vapor. Downstream of the membrane, the fluid is cooler and in the state of unsaturated vapor. The fluid either condenses at the upstream front of the membrane or a liquid film forms in front of the membrane. At an evaporation front within the membrane the fluid evaporates. The process depends on the permeability of the membrane and on surface tension.

INTRODUCTION

The Joule-Thomson process of a real gas is well known in thermodynamics. If a gas flows through a porous plug and there is a pressure loss associated with this process, but the change of kinetic energy is negligible, then the specific enthalpy of the gas is the same at an upstream and a downstream station. A gas with positive Joule-Thomson coefficient is cooler at the downstream station than at the upstream station. However, a saturated vapor with positive Joule-Thomson coefficient cannot simply cool down to reach the downstream state of unsaturated, cooler vapor. Instead, fronts of phase change occur. The process was analyzed previously neglecting capillary pressure [1, 2]. Here, the process is described assuming an ideally wetting liquid phase and accounting for the capillary pressure at a curved meniscus between the liquid and its vapor.

GOVERNING EQUATIONS

The governing equations consist of the balances of mass, momentum, and energy, respectively,

$$\dot m = const.,$$
$$\dot m = -\frac{\kappa}{\nu}\frac{\mathrm{d}p}{\mathrm{d}z},$$
$$\dot m h + \dot q = const.,$$

supplemented by Fourier's law of heat conduction and the thermic and caloric equations of state for the fluid. Above, $\dot m$ denotes the mass flux, κ is the permeability, z is the coordinate pointing in flow direction and $\dot q$ refers to the heat flux. The effective kinematic viscosity and the specific enthalpy of the fluid are given by ν and h, respectively, where the fluid can be a single phase or a two-phase mixture. The pressure difference between liquid and vapor at the evaporation front within the membrane is given by the capillary pressure,

$$\Delta p_{\mathrm{cap}} = \sigma \Big/ \sqrt{3\kappa/\varepsilon}\,,$$

where σ is the surface tension and ε refers to the porosity. At the membrane front the curvature of the menisci is undetermined, therefore the pressure difference can aquire any value between 0 and Δp_{cap}. As boundary conditions upstreams the state of the fluid and downstreams the pressure are given. The upstream state is specified by the temperature and the vapor mass fraction being equal to one. Also, the pressure must be equal to the saturation pressure at the given temperature.

RESULTS

In a previous study [1] it was found that the process depends on the permeability of the membrane with respect to a critical permeability,

$$\kappa_{\mathrm{c}} = \nu_{\mathrm{l}} T_1 (v_{\mathrm{g}} - v_{\mathrm{l}}) k_{\mathrm{l}} / \Delta h_{\mathrm{lg}}^2,$$

where ν_{l} is the kinematic viscosity of the liquid, v_{g} and v_{l} are the specific volumes of the gaseous and the liquid phase, respectively, k_{l} denotes the thermal conductivity of the fluid-filled porous material, T_1 is the absolute temperature upstream of the membrane and Δh_{lg} is the specific evaporation enthalpy.

If the permeability of the membrane lies above the critical value, $\kappa > \kappa_{\mathrm{c}}$, then the saturated vapor condenses partially at the upstream front of the membrane [1, 2]. Due to the condensation it releases heat that is conducted in downstream direction. A two-phase mixture flows through the membrane under decrease of pressure and temperature. The mixture fully evaporates at an evaporation front within the membrane, consuming the heat conducted downstreams. The vapor continues to flow in downstream direction under decrease of pressure, but with nearly constant temperature. The vapor undersaturates and reaches the final, unsaturated state at the downstream front of the membrane. Since the volume fraction of vapor in the two-phase mixture is close to unity even for a very small mass fraction, the capillary pressure between the region containing the two-phase mixture and the vapor is negligible.

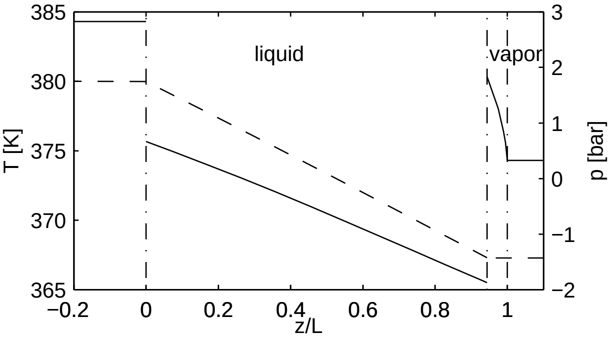

Figure 1. Temperature and pressure distribution for $\kappa/\kappa_c = 0.5$. Solid line: p, dashed line: T, dash-dotted lines: Fronts of phase change and membrane fronts.

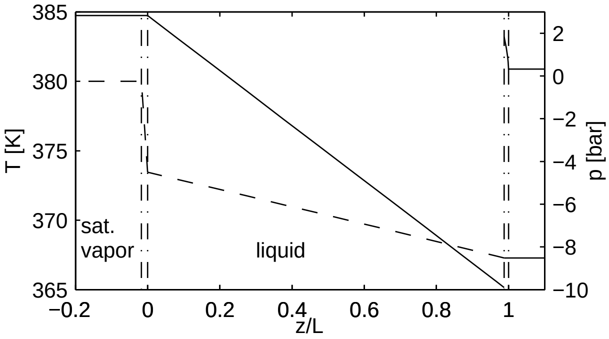

Figure 2. Temperature and pressure distribution for $\kappa/\kappa_c = 0.05$. For definitions of lines see adjacent figure.

If the permeability of the membrane lies below κ_c, the fluid condenses fully. For $\kappa \geq \kappa_f$ the condensation front is located at the upstream front of the membrane. The liquid menisci in the pores of the membrane maintain a pressure difference between the region upstream of the membrane and the liquid in the membrane. For $\kappa < \kappa_f$ a liquid film forms in front of the membrane. The pressure in the film is the same as upstreams of the membrane. The ratio κ_f/κ_c is given by

$$\kappa_f/\kappa_c = 1 - \tfrac{1}{2}\mathrm{Ca}_{\mathrm{JT}}^{-2}\left(\sqrt{1+4\mathrm{Ca}_{\mathrm{JT}}^{2}} - 1\right), \tag{1}$$

where $\mathrm{Ca}_{\mathrm{JT}}$ denotes a capillary number,

$$\mathrm{Ca}_{\mathrm{JT}} = \mu_{\mathrm{JT}}\frac{\mathrm{d}p_s}{\mathrm{d}T}\frac{\Delta p_{12}}{\Delta p_{\mathrm{cap,c}}},$$

evaluated at the upstream temperature T_1. Here, μ_{JT} is the Joule-Thomson coefficient, $\mu_{\mathrm{JT}} = (\partial T/\partial p)_h$, p_s is the saturation pressure, Δp_{12} is the pressure difference applied to the membrane and $\Delta p_{\mathrm{cap,c}}$ denotes the capillary pressure, calculated with the effective pore radius at the critical permeability.

Pressure and temperature distributions for $\kappa_f < \kappa < \kappa_c$ and $\kappa < \kappa_f$ are shown in Figs. 1 and 2, respectively. A system consisting of ethanol and a ceramic membrane with a thermal conductivity $k_m = 23\,\mathrm{W/mK}$ is investigated. The upstream state of the fluid is given by $T_1 = 380\,\mathrm{K}$ and $p_1 = p_s(T_1) = 2.83\,\mathrm{bar}$. A pressure difference of $\Delta p_{12} = 2.5\,\mathrm{bar}$ is applied. For the conditions under consideration, Eq. (1) yields $\kappa_f/\kappa_c = 0.104$.

For $\kappa = 0.5\kappa_c$, see Fig. 1, the condensation front is located at the upstream front of the membrane, $z = 0$. The pressure difference across the condensation front is approximately equal to half the capillary pressure. Liquid permeates through the membrane under decrease of pressure and temperature. At the evaporation front, located close to the downstream front of the membrane, the fluid evaporates. The pressure increases by an amount equal to the capillary pressure. In the remaining part of the membrane the vapor undersaturates nearly isothermically.

For $\kappa = 0.05\kappa_c$ a thin liquid film is formed, see Fig. 2. In the film the pressure remains constant and equal to its upstream value while the temperature decreases. In the liquid flow region in the membrane pressure and temperature decrease. The pressure difference at the evaporation front is much larger than in the previous example because the permeability and hence the pore size is smaller here. Note, that the pressure attains large negative values. This is consistent with experimental observations of a liquid bridge with a concave meniscus of large curvature [3].

CONCLUSIONS

The Joule-Thomson process of a fluid undergoing phase changes is investigated. Extending previous studies [1, 2], the capillary pressure is taken into account. The process depends on the permeability of the membrane with respect to the critical permeability κ_c and on a capillary number $\mathrm{Ca}_{\mathrm{JT}}$. Preliminary computations were performed for a fluid with an ideally wetting liquid phase. It is shown that the formation of a liquid film only occurs for $\kappa < \kappa_f$, a value which is much smaller than κ_c.

References

[1] Schneider W.: Vapor flow through a porous membrane — a throttling process with condensation and evaporation. *Acta Mech.* **47**:15–25, 1983.

[2] Loimer T.: A Joule-Thomson process with condensation and evaporation. *Proc. Appl. Math. Mech.* **3**:390–391, 2003.

[3] Fisher L.R., Israelachvili J.N.: Experimental studies on the applicability of the Kelvin equation to highly curved concave menisci. *J. Colloid Interface Sci.* **81**:528–541, 1981.