

Experimental study of a supersonic mixing layer with an estimation of acoustic radiation

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Abstract

The study of turbulent compressible flow is the object of a particular interest in this last decade. The striking attract is that many phenomena become very important with the effect of compressibility. Indeed, the structure of turbulence changes drastically and depends essentially on the convective Mach number M_c , i.e. the Mach number based on the velocity difference between the large eddies and the external flows. One flow where the effect of compressibility on turbulence appears at relatively low Mach number is the plane mixing layer.

The study of a supersonic mixing layer on self preservation was investigated in this paper. In order to accomplish this investigation we have got a large test section in a wind tunnel. The setup of an isobaric mixing layer was obtained by a judicious downstream conditions in the diffuser. The mean field profiles were measured and correspond to similarity solutions. The estimation of the acoustic radiation shows that this term is insignificant against the production.

Keywords: acoustic radiation, mixing layer, convective Mach number.

1 Introduction

The study of compressible mixing layer has its importance both from technological point of view and scientific one. Indeed, with the great interest given to phenomena related to the effect of compressibility on turbulence, and on account of the importance of this type of flow in many applications (supersonic combustion chamber, noise reduction of supersonic jet), many

fundamental studies, either theoretical or experimental, were carried out in this last decade on compressible turbulence in general, and on compressible mixing layer in particular. However, some mean characteristics as the shape of velocity profiles or the level of turbulence have not received much attention. In some experimental papers (Ikawa and Kubota [1], Elliot and Samimy [2] for example), it has been noticed that the shape of the mean velocity profiles in subsonic and supersonic mixing layer looks very much the same. This was corroborated by the numerical work of Zeman [3] for high speed mixing layers with constant density. As shown by Papamoschou and Roshko [4], the spreading rate depends on Mach number, and is, in away, independent of transverse density gradients. Considerable efforts have been made in the last decade to explain this effect, but the reason of such a decrease is still not totally identified. Verman et al. [5] have proposed to relate it to a decrease in the production of turbulence. There are several other likely candidates to explain the observed effects, such as a modification of the anisotropy of the Reynolds stress related to an alteration of the structure of pressure, an enhancement of dissipation through dilatation dissipation in Mach wave or shock waves produced by eddies, or the loss of the energy by acoustic radiation. However, none of these hypotheses has been conclusively checked by experiment, and results on this side are still badly needed.

2 Experimental setup

The experiments were conducted in the continuous wind tunnel of the Institut de Recherche sur les Phénomènes hors Equilibre (*IRPHE*), Marseille. In the outer supersonic flow, the stagnation pressure was $P_{te}=0.5\text{atm}$, the stagnation temperature was $T_{te}=300\text{K}$. The test section is rectangular: 12cm wide and 15cm high. The precedent researches (Quine [6], Barre [7]) have shown that in the last sections of the setup the mixing layer is not self-similar. So a new test section more long have permitted to extend the explorations up to distance of 300mm down stream of the splitting plate. Moreover, modifications were made to displacement of the probe and the junction between the test section and the diffuser. To check the modification of the previous flow, we have controlled the flow with Schlieren visualisation. Furthermore, to control the mean flow, we have performed a longitudinal exploration of the static pressure in the outer flows and we have measured the pressure on the inferior wall of the test section. We know that the isobary of the layer depends sensibly on the downstream conditions in the diffuser, so a particular attention was brought to examine this point.

The mixing layer was installed with mixing two independently controlled streams separated by a splitter plate (fig. 1).

The boundary layer of the high velocity side is turbulent and fully developed. The measurements presented here were carried out up to longitudinal distance of 300mm from the trailing edge of the splitter plate, i.e. 34 times the thickness of the initial high speed boundary layer.

The exploration of the mean field was performed with the same conventional methods[8](Measurements of static and pitot pressure and of stagnation temperature by thermocouple).

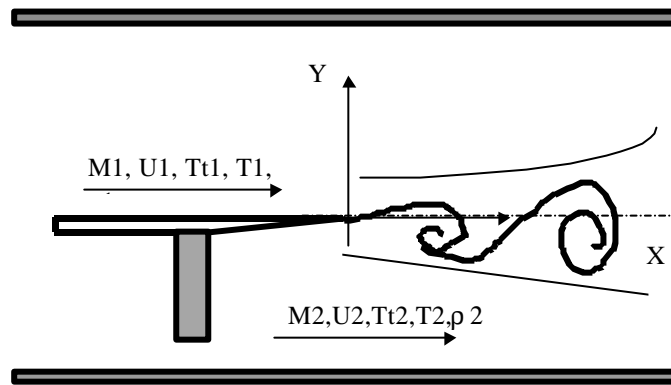


Figure 1: Sketch of a compressible mixing layer

3 Experimental results and discussion

The principal characteristics of the external mean flows are given in table 1.

Table 1: The principal characteristics of the outer streams

Flow	M	U(m/s)	T_t (K)	T(K)	ρ (kg/m ³)
High speed flow	1.83	490	298	178.5	0.20
Low speed flow	0.38	129	295.4	287.14	0.13

The static pressure profiles are presented in figure 2. We noticed that the pressure variations in a section are low, at least 5%. Furthermore, the longitudinal variation of the pressure is inferior to 5%.

The figure 3 shows total temperature profiles. We found in the high speed side a maximum which is typical to adiabatic boundary layers with a Prandtl number <1.

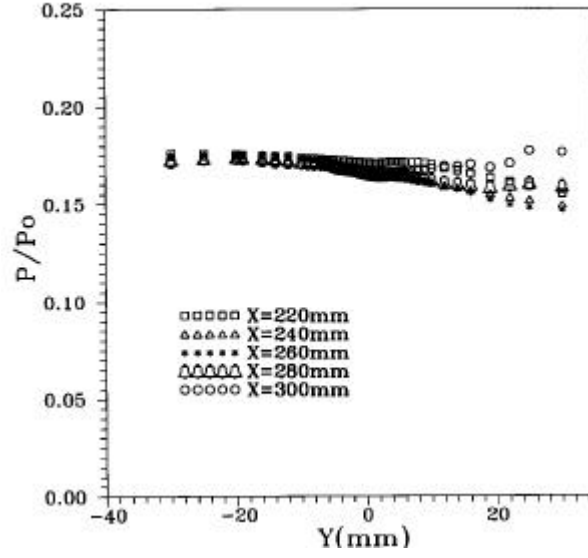


Figure 2: Static pressure profiles

The velocity profiles are affected by the initial supersonic boundary layer. So we have studied the characteristics of the initial boundary layer. The table 2 summarised the principal characteristics of the initial supersonic boundary layer.

Table 2: The initial boundary layer characteristics

M_e	d (mm)	\bar{d} (mm)	q (mm)	H	C_f	u_t (m/s)	t_w (N/m ²)
1.81	9	1.83	0.73	2.5	$2 \cdot 10^{-3}$	19.2	50.54

The displacement thickness \bar{d} is defined as:

$$\bar{d} = \int_0^{\infty} \left(1 - \frac{rU}{r_e U_e} \right) dy \quad (1)$$

The momentum thickness is defined as:

$$q = \int_0^{\infty} \frac{rU}{r_e U_e} \left(1 - \frac{U}{U_e} \right) dy \quad (2)$$

The shape factor H is:

$$H = \frac{d^*}{q} \quad (3)$$

The shear velocity u_t is related to wall friction τ_w by:

$$\tau_w = \rho_w u_t^2 \quad (4)$$

The mean velocity profiles in different sections are given in figure 4. The shape of profiles indicate an evolution from a boundary layer ($X=-5\text{mm}$) to a mixing layer ($X>160\text{mm}$). We can observe that the wake of the splitter plate disappear rapidly because the velocity difference between the two streams is important ($\Delta U=361\text{ m/s}$). A collapse of the velocity profiles showing similarity is observed for $X>160\text{mm}$.

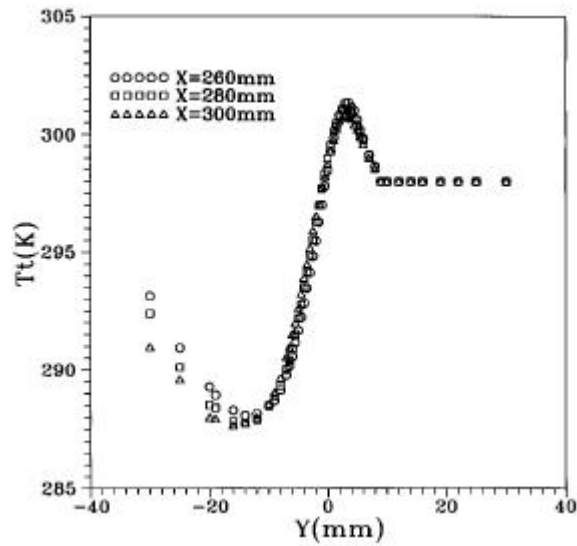


Figure 3: Total temperature profiles

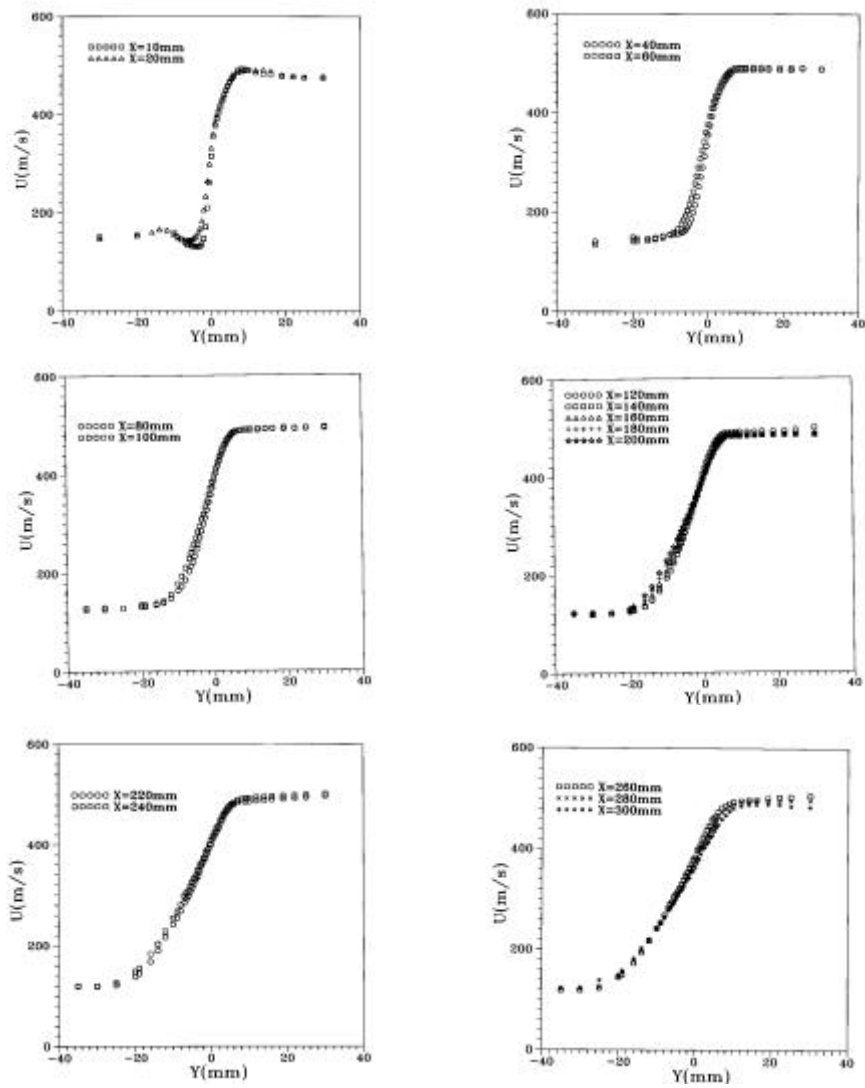


Figure 4 : mean velocity profiles

The spatial growth of the the layer is shown in figure 5. Two definitions of the thickness are used : the definition of the thickness δ given at the Stanford conference(the thickness is defined between the points where $U=U_2+0.95(U_1-U_2)$ and $U=U_2+0.316(U_1-U_2)$) and the vorticity thickness $\delta_\omega = \Delta U / (\partial U / \partial y)_{\max}$. The solid line is obtained by assuming that for a subsonic

layer with zero velocity ratio and unity density ratio, $(d\delta_s/dx)_0=0.115$. The formula proposed by Papamoschou and Roshko[4] to drive spreading rate of a layer with velocity ratio $r=U_2/U_1$, density ratio $s= \rho_2/\rho_1$ and convective Mach number M_c was employed:

$$\frac{d\delta}{dx} = \frac{1}{2} \left(\frac{d\delta}{dx} \right)_0 \frac{(1+r)(1+\sqrt{s})}{(1+r\sqrt{s})} f(M_c) \quad (5)$$

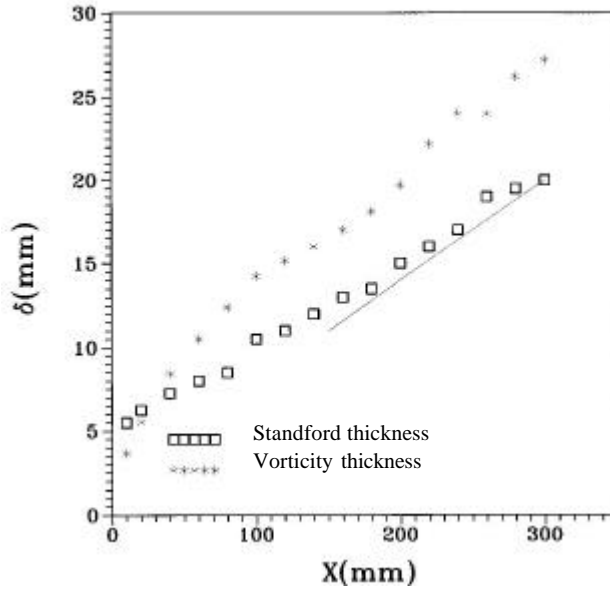


Figure 5: Longitudinal evolution of thickness

The definition of $f(M_c)$ is the same as in Papamoschou and Roshko[4]. The value of $f(M_c)$ used in the present work are the values reassessed in Smits and Dussauge[9]. The agreement with measurements is very good, and a linear growth is observed in the sections in which the mean velocity profile are self-similar.

3 Acoustic radiation

A key point for compressible turbulent flows is the importance of the energy loss by acoustic radiation. Barre et al [10] have measured the intensity of the pressure

radiated by their mixing layer into the supersonic external flow; they also measured the speed of the sound sources. However, they have not evaluated the energy radiated by acoustic waves, although their data can provide an assessment of the importance of such phenomena, as J. Laufer[11] did for turbulent boundary layers. It was checked in Muscat[12] that the pressure fluctuations in the outer flow in our configuration are in good agreement with the measurements of Barre et al.[10]. It is deduced that these results can be used for evaluating the acoustic losses in the mixing layer at a nominal Mach number of 0.61. The measurements have shown that the velocity difference $U_j - U_s$ between supersonic stream and the noise sources is supersonic. Hence, it can be assumed the pressure waves are predominantly Mach waves, and the characteristics of the energy flux can be derived from the usual relations for isentropic fluctuations. The energy flux per unit surface is usually called the acoustic intensity. This vector quantity is normal to the wave front, and its modulus is:

$$I = \frac{\overline{p'^2}}{\mathbf{r}a} \quad (6)$$

A question is to determine to which turbulent quantity the acoustic intensity should be compared. The simplest is probably to consider the equation for turbulent kinetic energy, and to integrate it over a control volume defined by two sections S_1 and S_2 , two horizontal planes H_1 and H_2 ; its dimension is unity in the spanwise direction .

The equation of the turbulent kinetic energy reads, for Favre averaged variables:

$$\frac{\partial(\overline{\mathbf{r}} \tilde{k} \tilde{U}_j)}{\partial x_j} = P + D - \overline{\mathbf{r}} \tilde{\mathbf{e}} \quad (7)$$

where $\tilde{k} = \frac{1}{2} \frac{\overline{\mathbf{r} u_j' u_j'}}{\overline{\mathbf{r}}}$, \tilde{U}_j are the components of the Favre averaged velocity, $\overline{\mathbf{r}}$ is the mean density, the overbar denotes an average, P is the

turbulent production, $D = \frac{\partial(\overline{\mathbf{r} u_i' u_i' u_j'} + \overline{p' u_j'})}{\partial x_j}$ is the term of turbulent

diffusion, $\tilde{\mathbf{e}}$ the rate of dissipation per unit mass, p' is the pressure fluctuation, and u_j' the components of velocity fluctuation. Integrating over the volume V bounded by the surface S yields:

$$\int_V \left(\frac{\partial(\overline{\mathbf{r}} \tilde{k} \tilde{U}_j)}{\partial x_j} \right) dV = \int_V P dV - \int_S (\overline{\mathbf{r} u_i' u_i' u_j'} + \overline{p' u_j'}) n_j dS - \int_V \overline{\mathbf{r}} \tilde{\mathbf{e}} \quad (8)$$

As we are interested by the energy flux radiated into the outer flows, we will consider only surface H_1 and H_2 . We calculate now the flux through H_1 , whose normal vector reduces to n_2 . As H_1 is chosen far from the edge of the mixing layer, the fluctuations are only of acoustic nature. The power involved by acoustic emission is estimated as:

$$P_a = 2 \frac{\overline{p'^2}}{ra} \frac{\sqrt{M_s^2 - 1}}{M_s} \quad (9)$$

Using the measurements of Barre et al.[10] in which $M_s = 1.4$ and $\frac{\overline{p'^2}}{p^2} = 4.10^{-4}$, it is found that $P_a = 1100 \text{W/m}^2$, while the integral of production is 3.2×10^4 . Therefore, the acoustic loss of energy is only 3% of the production term. The level is of course weak.

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