

## MECHANICS OF THE BOUNDING FLIGHT REVISITED

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**Summary** Bounding flight, the technique used by many small birds, is studied by control theory and rigorous argument in terms of time averages. Two major conclusions are obtained: the bounding flight is considered as the optimal bang-bang control for the unstable back-side flight; the bounding flight saves more energy than the continuous powered-flight in wide speed ranges, in particular at low speeds, and hence that the formerly known speed bound is found to be insufficient to account for the phenomenon.

### INTRODUCTION

Many birds fly with a technique by mixing wing-flapping and wing-folding phases. Such a flight trajectory undulates up and down. This is the bounding flight. The former studies showed the lower bound of flight velocity suited for the bounding flight [1-3]. Most researchers in biomechanics reached the same conclusion. But biologists did not admit the theory, because birds fly intermittently in almost all the speed ranges. Even now there are two hypotheses proposed: the fixed-gear hypothesis and the body-lift hypothesis. The present study affords a strong support to the former hypothesis from a mechanical viewpoint. In the extended summary we use nomenclatures without notice if those are conventional.

### THEORY

#### Basic equations

We take  $x$ - and  $z$ -axes parallel and perpendicular to the horizon, respectively. We assume a bird with its mass  $m$  flying at the speed  $u$  toward the positive  $x$  direction. Let us define the fraction of the flapping duration  $t_p$  to the total flight-cycle time  $T$  by  $f = t_p/T$ . To be aloft, it is necessary  $mg = f^{-1} \rho \langle u^2 \rangle SC_L/2$  for  $t$  in  $[0, t_p]$ , where  $\langle \rangle$  denotes a time-averaged value over one cycle. But the analysis becomes concise if we adopt the following sufficient condition:  $mg = f^{-1} \rho u^2 SC_L/2$  for  $t$  in  $[0, t_p]$ . Then the equation of vertical motion assumes the form:

$$\ddot{z} = \begin{cases} (f^{-1} - 1)g & \text{for } t \in [0, t_p] \\ -g & \text{for } t \in [t_p, T] \end{cases}, \quad \text{while } \dot{u} = \begin{cases} \tau - (A_{w0} + A_b)u^2 - A_{w1}u^{-2} & \text{for } t \in [0, t_p] \\ -A_b u^2 & \text{for } t \in [t_p, T] \end{cases}$$

corresponds to the equation of horizontal

motion. In the equations shown above,  $\tau$  and  $A$ -something respectively denote the acceleration due to the thrust and aerodynamic coefficients, while the subscripts  $w$  and  $b$  designate wing and body, respectively. Fig.1 shows the typical phase portrait of the bounding flight for a cycle. We used Tennekes's data of a White Wagtail [4]. A bird starts flapping at  $t = 0$  and  $u = u_{min}$  till  $t = t_p$  and  $u = u_{max}$ , then it switches off till  $t = T$  and  $u = u_{min}$ . This is one cycle.

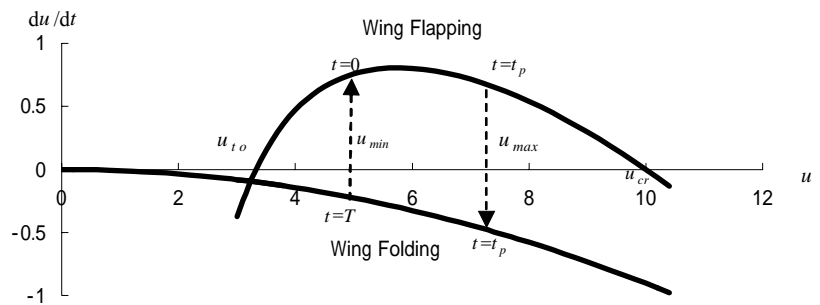


Fig.1. Phase portrait of the bounding flight

### RESULTS AND DISCUSSION

#### Implication of the bounding flight as optimum control

Flapping flight with the minimal constant thrust is attained at  $u = u_{opt} [A_{w1}/(A_{w0} + A_b)]^{1/4}$  as  $\tau = \tau_{opt} 2[(A_{w0} + A_b)A_{w1}]^{1/2}$ . The stationary solution becomes unstable with the constant thrust if the target cruising speed  $u^*$  is less than  $u_{opt}$ . This situation is called back side. Then it becomes necessary to control flight conditions. We shall consider the thrust as a control variable and formulate the problem as the optimum regulator:

$$\text{Minimize } \int_0^T (u - u^*)^2 dt,$$

subject to the equality constraint, *i.e.*, the equation of horizontal motion, and the inequality constraint on the thrust:  $0 \leq \tau \leq \tau_{max}$ . Since this regulator is linear in terms of  $\tau$ , the solution is given by the well-known bang-bang control:

$$\tau = \begin{cases} \tau_{max} & \text{if } \partial J / \partial u > 0, \\ 0 & \text{otherwise,} \end{cases}$$

where  $J$  denotes the objective functional above. This corresponds exactly to the bounding flight.

### Derivation of the exact time-averaged values and annotation of the results

The former studies relied solely on the analysis of instantaneous powers, for this formalism needs simple algebra only. This simplification, however, misled us to the insufficient conclusion declined by biologists.

Let us consider the bounding flight with the speed range  $u$  in  $[u_{min}, u_{max}]$ . The time-averaged flight velocity  $\langle u \rangle$  can be calculated by integrating the equation of horizontal motion:

$$\langle u \rangle = \frac{u_{cr}^2 \ln \left| \frac{u_{cr}^2 - u_{min}^2}{u_{cr}^2 - u_{max}^2} \right| + u_{to}^2 \ln \left| \frac{u_{max}^2 - u_{to}^2}{u_{min}^2 - u_{to}^2} \right| + \frac{2\sqrt{\tau^2 - 4(A_{w0} + A_b)A_{w1}f^{-1}}}{A_b} \ln \left| \frac{u_{max}}{u_{min}} \right|}{u_{cr} \ln \left| \frac{u_{cr} + u_{max}}{u_{cr} - u_{max}} \right| \left| \frac{u_{cr} - u_{min}}{u_{cr} + u_{min}} \right| + u_{to} \ln \left| \frac{u_{max} - u_{to}}{u_{max} + u_{to}} \right| \left| \frac{u_{min} + u_{to}}{u_{min} - u_{to}} \right| + \frac{2\sqrt{\tau^2 - 4(A_{w0} + A_b)A_{w1}f^{-1}}}{A_b} \left( \frac{1}{u_{min}} - \frac{1}{u_{max}} \right)},$$

where  $u_{cr}$  and  $u_{to}$  denote the high- and low-speed stationary solutions, respectively. Using  $\langle u \rangle$ , we obtain the required specific-power for the bounding flight  $P_b = \tau_{max} \langle u \rangle$ . On the other hand in the continuous powered-flight both the thrust and the flight velocity are constant. To fly at  $u = \langle u \rangle$  the thrust has to be adjusted such that  $\tau = (A_{w0} + A_b) \langle u \rangle^2 + A_{w1} \langle u \rangle^{-2}$ . Hence we have the required specific-power for the continuous powered-flight  $P_c = [(A_{w0} + A_b) \langle u \rangle^3 + A_{w1} \langle u \rangle^{-1}] T$ .

The scheme to solve the entire problem starts from giving  $u_{min}$  and  $u_{max}$ , and then we shall calculate the corresponding fraction  $f$  and finally  $\langle u \rangle$ . Fig.2 shows  $\langle u \rangle$ -contours in  $[u_{min}, u_{max}]$  plane. Fig.3 shows  $f$ -contour in  $[u_{min}, u_{max}]$  plane. The darker the gradation becomes, the higher those calculated values become. Roughly speaking, the mean velocity  $\langle u \rangle$  is close to the arithmetic mean between  $u_{min}$  and  $u_{max}$ . The fraction  $f$  has the minimum value, which assures the physically meaningful solution and depends on  $A_{w0}$ ,  $A_{w1}$ ,  $A_b$ ,  $\tau_{max}$ ,  $u_{min}$  and  $u_{max}$ . In the present case the minimum ranges from around 0.6. Fig.4 summarizes the essence of our findings:  $P_b - P_c$  contour in  $[u_{min}, u_{max}]$  plane; red and blue denote negative and positive, respectively. The diagonal part, in other words  $u_{min} \approx u_{max}$ , is the most likely flight region, where the bounding flight saves more energy than the continuous powered-flight in wide speed ranges, in particular at low speeds. Fig.5 is the White Wagtail's typical flight path reconstructed by solving the equations of motion:  $T = 1.4$  s,  $f = 0.62$ ,  $\langle u \rangle = 6.5$  m/s with  $u_{min} = 6.4$  m/s and  $u_{max} = 6.6$  m/s. The variation in altitude is around 1 m during the bird flies horizontally 8.5 m ahead.

### CONCLUSIONS

The mathematically rigorous analysis leads us to the conclusions: the bounding flight is the optimal bang-bang control; this technique saves more energy than the continuous powered-flight in wide speed ranges, in particular at low speeds.

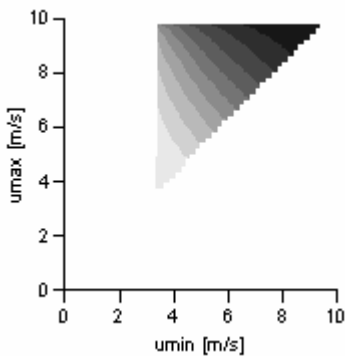


Fig.2.  $\langle u \rangle$ -contour in  $[u_{min}, u_{max}]$  plane.

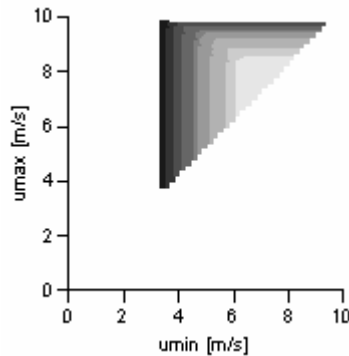


Fig.3.  $f$ -contour in  $[u_{min}, u_{max}]$  plane.

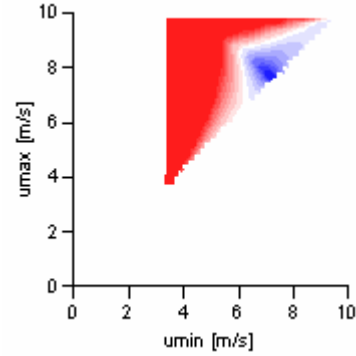


Fig.4.  $P_b - P_c$  contour in  $[u_{min}, u_{max}]$  plane.

### References

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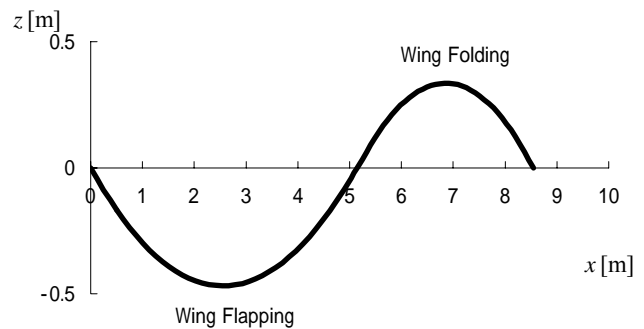


Fig.5. Reconstructed flight path of a White Wagtail.