Hydrodynamic interactions between particles on a flat free-surface

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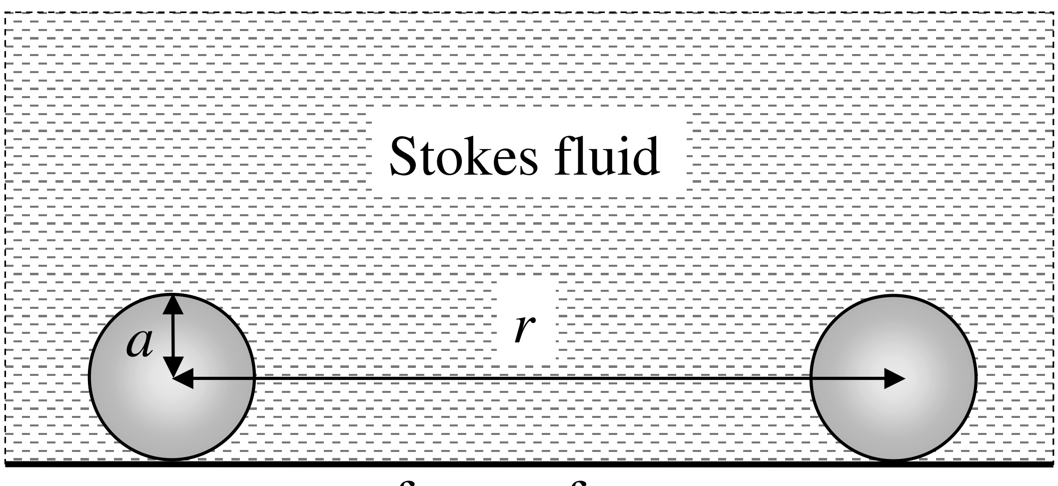
J. Chem. Phys. 2004, Europhys. Lett. 2004

QUASI-TWO-DIMENSIONAL SYSTEM



Experiments, G. Maret et al.

GOAL

• Find the Q2D hydrodynamic interactions.
(For given forces and torques applied to the particles, determine their velocities.)

• Construct analytic approximation to Q2D mobility matrix, valid for large interparticle distances.

THEORETICAL BACKGROUND

Assumptions.

• Small Reynolds number.
• Stokes equations,

$$\eta \boldsymbol{\nabla}^2 \mathbf{v} - \boldsymbol{\nabla} p = \mathbf{0}, \qquad \boldsymbol{\nabla} \cdot \mathbf{v} = 0.$$

• Stick boundary conditions on the surfaces of the spheres.
• Flat free surface.

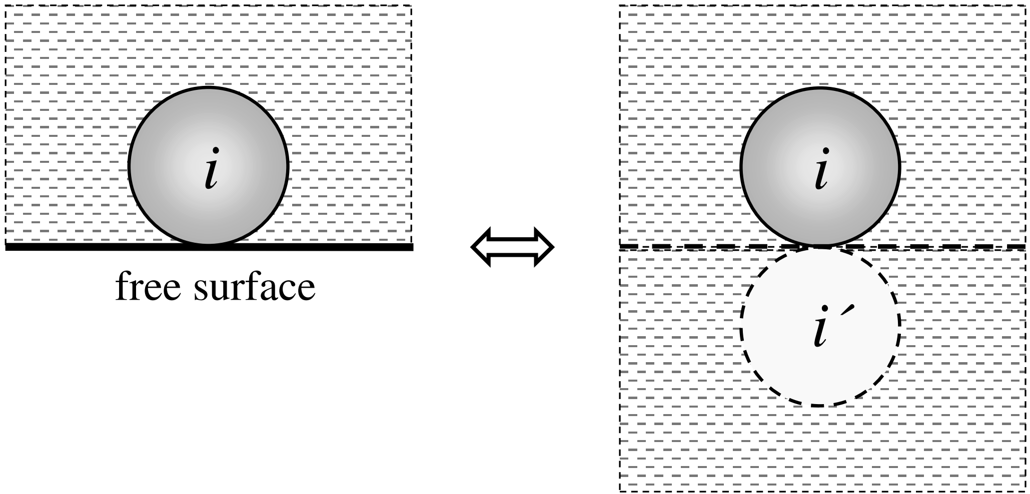
3D hydrodynamic interactions (HI).

• Green tensor (Oseen).
• Boundary integral equation,

$$\mathbf{v}(\mathbf{r}) = -\int_S \mathbf{T}_0(\mathbf{r} - \bar{\mathbf{r}}) \cdot \boldsymbol{\sigma}(\bar{\mathbf{r}}) \cdot \mathbf{n}\, dS, \qquad \mathbf{r}, \bar{\mathbf{r}} \in S.$$

• Multipole expansion.
• 3D mobility matrix for a given configuration of spheres.
• Properties of HI - non-additive, long-range, lubrication.

THEORETICAL METHOD – IMAGES



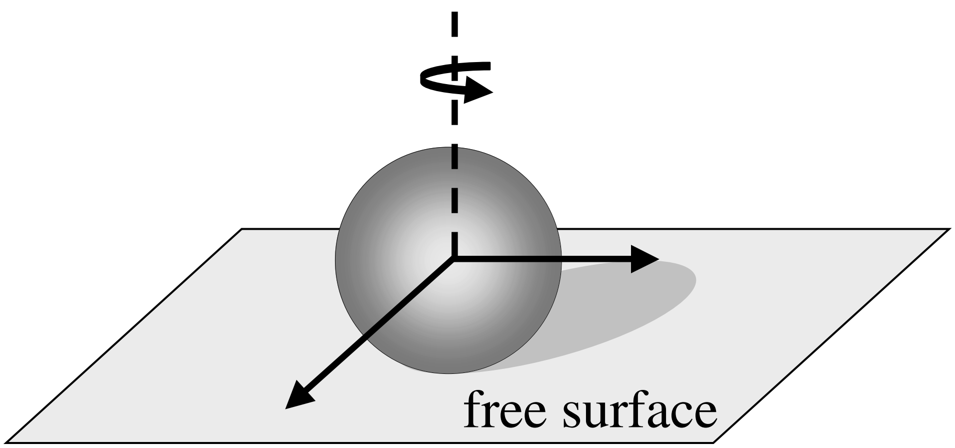
$\mathcal{R}_F$ – reflection with respect to the free surface at $z = 0$

Green tensor, $$\mathbf{T}_F(\boldsymbol{r}, \bar{\boldsymbol{r}}) \;=\; \mathbf{T}_0(\boldsymbol{r} - \bar{\boldsymbol{r}}) + \mathbf{T}_0(\boldsymbol{r} - \bar{\boldsymbol{r}}') \cdot \mathcal{R}_F$$

LUBRICATION

Difficulty – almost all matrix elements of the one-sphere grand resistance operator are infinite.

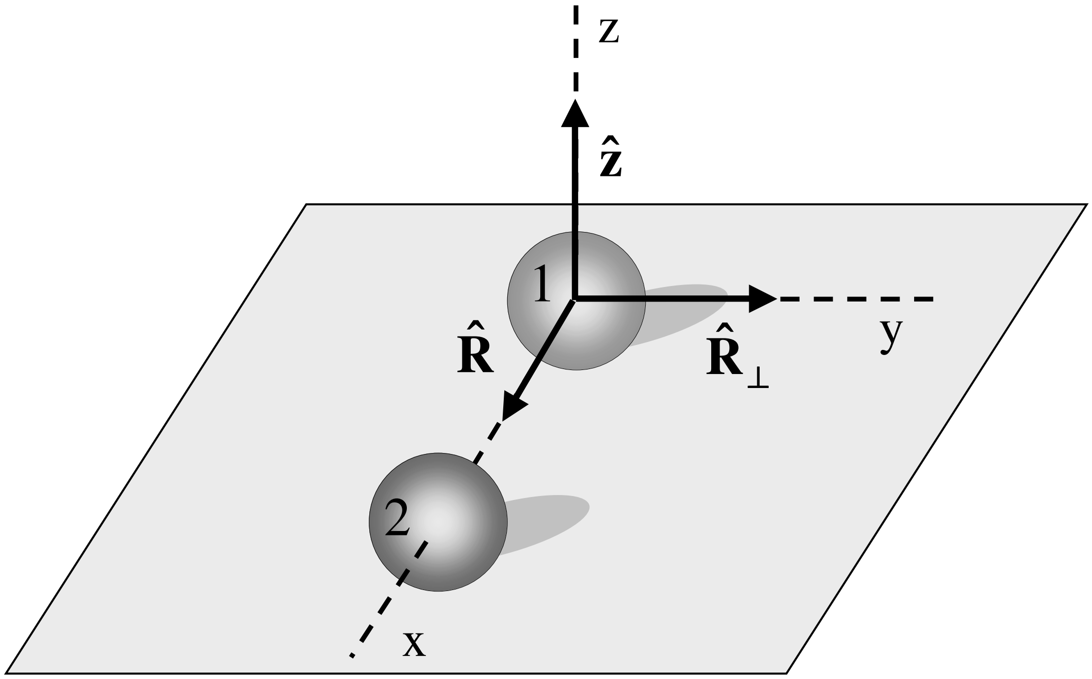
Proper treatment – combinations of the multipole functions symmetric with respect to reflections.



Three degrees of freedom of a sphere, which touches the planar free surface.

Q2D MOBILITY

Two spheres



$$\begin{pmatrix} U_{1x} \\ U_{1y} \\ \Omega_{1z} \end{pmatrix} = \boldsymbol{\mu}_{11} \begin{pmatrix} F_{1x} \\ F_{1y} \\ T_{1z} \end{pmatrix} + \boldsymbol{\mu}_{12} \begin{pmatrix} F_{2x} \\ F_{2y} \\ T_{2z} \end{pmatrix} .$$

$$U_{iz} = 0, \quad \Omega_{ix} = \Omega_{iy} = 0,$$

Many spheres

large interparticle distances $R \rightarrow$ pairwise approximation,

$$\boldsymbol{\mu}_{12}(1...N) = \boldsymbol{\mu}_{12}(12), \qquad \boldsymbol{\mu}_{11}(1...N) = \boldsymbol{\mu}(1).$$

RESULTS – Q2D SELF-MOBILITY

$$\boldsymbol{\mu}_{11} = \frac{1}{6\pi\eta a}\begin{pmatrix} 1.3799554 & 0 & 0 \\ 0 & 1.3799554 & 0 \\ 0 & 0 & 1.10920983 \end{pmatrix} + o(\frac{1}{R^3}).$$

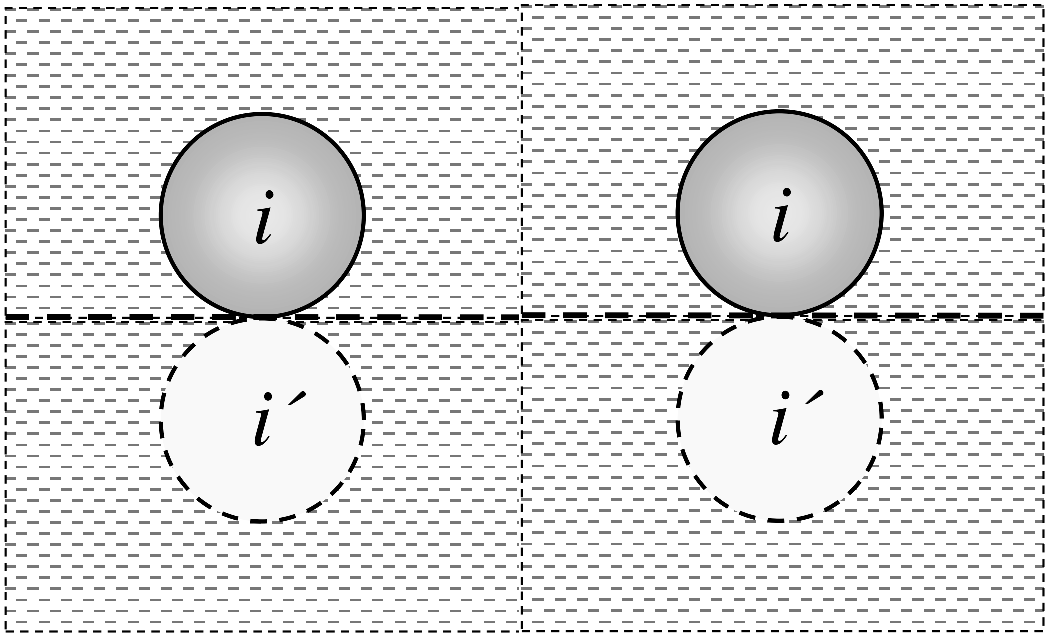
RESULTS – Q2D DISTINCT-MOBILITY

$$\boldsymbol{\mu}_{12} = \frac{1}{6\pi\eta a}\left[\begin{pmatrix} \frac{3}{2} & 0 & 0 \\ 0 & \frac{3}{4} & 0 \\ 0 & 0 & 0 \end{pmatrix}\frac{1}{R} - \begin{pmatrix} 1.159862... & 0 & 0 \\ 0 & 0.111686... & 0 \\ 0 & 0 & 0 \end{pmatrix}\frac{1}{R^3}\right] + \frac{1}{4\pi\eta a^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}\frac{1}{4R^2} - \frac{1}{8\pi\eta a^3}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\frac{1}{8R^3} + o(1/R^3),$$

COMPARISON WITH 3D

- the leading Q2D terms = 2x larger than 3D,
- no simple relation between higher order Q2D and 3D terms,

$$\boldsymbol{\mu}_{12}^{tt(3D)} = \frac{1}{6\pi\eta a}\left[\begin{pmatrix} \frac{3}{4} & 0 \\ 0 & \frac{3}{8} \end{pmatrix}\frac{1}{R} - \begin{pmatrix} \frac{1}{8} & 0 \\ 0 & -\frac{1}{16} \end{pmatrix}\frac{1}{R^3}\right] + o(1/R^3),$$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CONCLUSIONS

• The scheme to calculate the hydrodynamic interactions for a quasi-two-dimensional system has been constructed.

• The long-distance leading terms of the two-sphere mobility matrix have been evaluated up to the order $\mathcal{O}(1/R^3)$.