Stability and Break-up of Jets of Viscoelastic Fluids

Kick-Off Meeting CONEX Project

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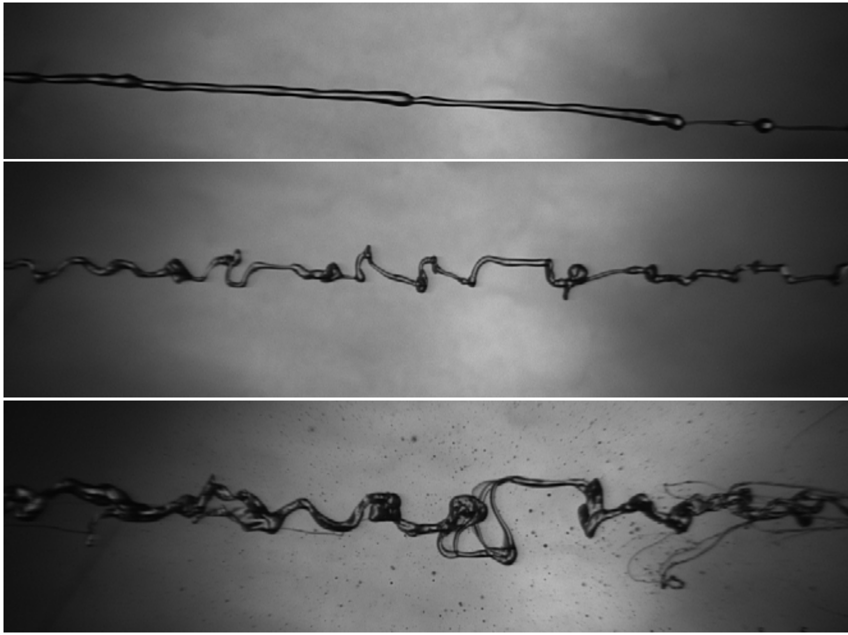
Motivation and Aims

- Break-up of viscoelastic liquid jets is an important process
- Stability behaviour is to be investigated
- Onset of different break-up mechanisms is of interest



→ Generalisation of the Ohnesorge jet break-up nomogram is attempted

Linear Stability Analysis - Basic Equations

Continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0$$

Momentum

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \rho \mathbf{v} = -\nabla \cdot \pi + \rho \mathbf{g}$$

where $\pi = p\delta + \tau$

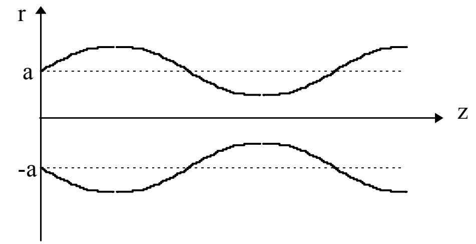
Rheological constitutive equation - Oldroyd model

$$\tau + \lambda_1 \frac{D\tau}{Dt} + \frac{1}{2}\mu_0 (tr\tau)\dot{\gamma} - \frac{1}{2}\mu_1 \{\tau \cdot \dot{\gamma} + \dot{\gamma} \cdot \tau\} + \frac{1}{2}\nu_1 (\tau : \dot{\gamma})\delta$$

$$= -\eta_0 [\dot{\gamma} + \lambda_2 \frac{D\dot{\gamma}}{Dt} - \mu_2 \{\dot{\gamma} \cdot \dot{\gamma}\} + \frac{1}{2}\nu_2 (\dot{\gamma} : \dot{\gamma})\delta]$$

(G. Brenn et al., IJMF vol. 26 (2000), 1621-1644)

Linearised Equations for Incompressible Fluid



Continuity $$\nabla \cdot \mathbf{v} = 0$$

Momentum $$\rho\left(\frac{\partial}{\partial t} + \overline{U}\frac{\partial}{\partial x}\right)\mathbf{v} = -\nabla \cdot (p\delta + \tau)$$

Rheology $$\tau + \lambda_1\left(\frac{\partial}{\partial t} + \overline{U}\frac{\partial}{\partial x}\right)\tau = -\eta_0\left[\dot{\gamma} + \lambda_2\left(\frac{\partial}{\partial t} + \overline{U}\frac{\partial}{\partial x}\right)\dot{\gamma}\right]$$

Boundary conditions - Linearised form (cylindrical coordinates)

$$v_r = \left(\frac{\partial}{\partial t} + \overline{U} \cdot \nabla\right)\xi \qquad v_r = \frac{\partial \xi}{\partial t} + \overline{U}\frac{\partial \xi}{\partial z} \qquad r = a$$

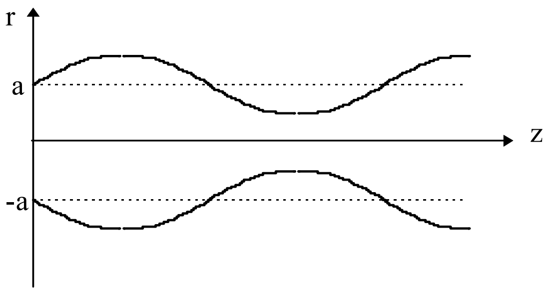
$$(\pi - \pi_g) \times \mathbf{n} = 0 \qquad \pi_{rz} = \tau_{rz} = -\eta(\alpha)\left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right) = 0 \qquad r = a$$

$$(\pi - \pi_g) \cdot \mathbf{n} + \sigma \nabla \cdot \mathbf{n} = 0 \qquad \pi_{rr} - \pi_{g,rr} + p_\sigma = 0 \qquad r = a$$

Wave Solution and Dispersion Relation

We seek solutions in the form of moving waves

$$\phi = \Phi(r)e^{ikz+\alpha t}$$



k - real wave number

α - complex frequency

real part - disturbance growth rate

imaginary part - frequency

The same procedure is applied to the gas phase also, but the gas treated as inviscid. Expressing the normal stresses in the liquid and in the gas in terms of the above results:

Dispersion relation emerges

Dispersion relation for viscoelastic jet

Disturbance growth rate as a function of the non-dimensional wavenumber

$$\alpha^2\left[\frac{ka}{2}\frac{I_0(ka)}{I_1(ka)}+\frac{ka}{2}\frac{\rho_g}{\rho}\frac{K_0(ka)}{K_1(ka)}\right]+$$

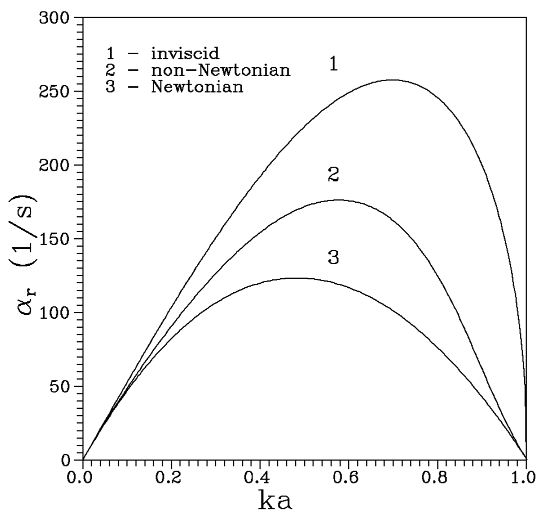
$$\alpha\left\{2ik\overline{U}_R\frac{ka}{2}\frac{\rho_g}{\rho}\frac{K_0(ka)}{K_1(ka)}+\frac{\eta_0}{\rho a^2}\frac{1+\lambda_2\left(\alpha+ik\overline{U}_R\right)}{1+\lambda_1\left(\alpha+ik\overline{U}_R\right)}k^2a^2\left[2ka\frac{I_0(ka)}{I_1(ka)}\frac{l^2}{l^2-k^2}-1-2la\frac{I_0(la)}{I_1(la)}\frac{k^2}{l^2-k^2}\right]\right\}$$

$$=\frac{\sigma}{2\rho a^3}k^2a^2\left(1-k^2a^2\right)+C\cdot\frac{\rho_g}{\rho}\frac{\overline{U}_R^2}{2a^2}k^3a^3\frac{K_0(ka)}{K_1(ka)}$$

frequency dependent viscosity

capillary influence

aerodynamic influence

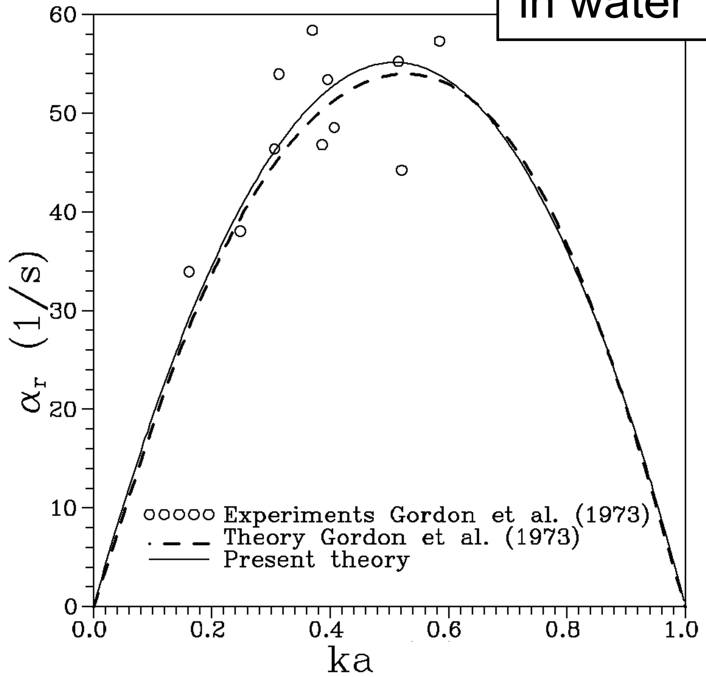


$\overline{U}_R$ = 2m/s	η_0 = 0.1Pa·s
a =500µm	λ_1 = 10ms
ρ =1000kg/m³	λ_2 = 1ms
ρ_g =1.2kg/m³	(Z=0.535,
σ =70mN/m	We=28.6)

(C = 0.175)

Validation of the Theory - I

500ppm Separan 30 in water



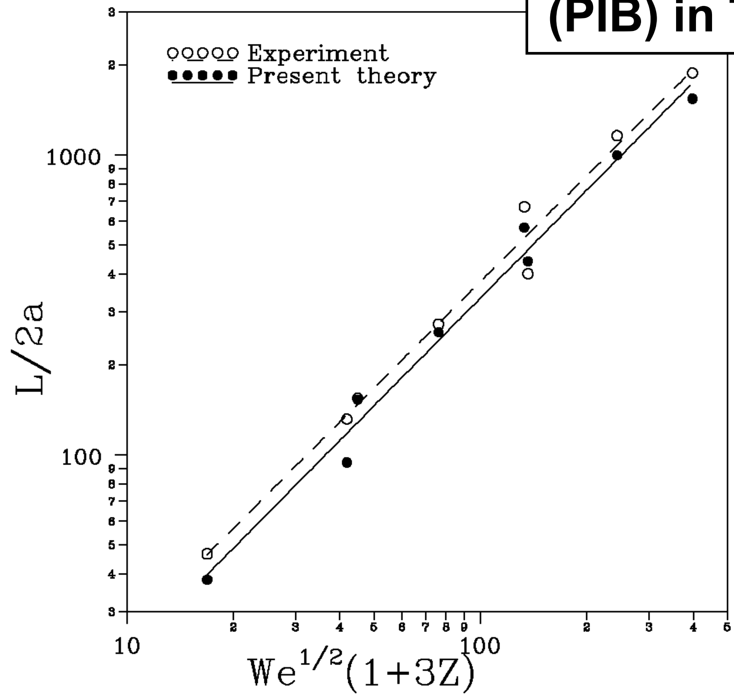
Aqueous polyacrylamide solution

$\overline{U}_R = 0$	$\eta_0 = 0.11 Pa \cdot s$
a =921µm	$\lambda_1 = 0.1 ms$
$\rho = 1000 kg/m^3$	$\lambda_2 = 0.01 ms$
$\sigma = 70 mN/m$	(Z=0.431)

Growth rate = f(ka) from present theory and experiments / theory by Gordon et al. (1973) - good agreement

Validation of the Theory - II

3.78% L100 (PIB) in Tetralin



Organic polyisobutylene solution

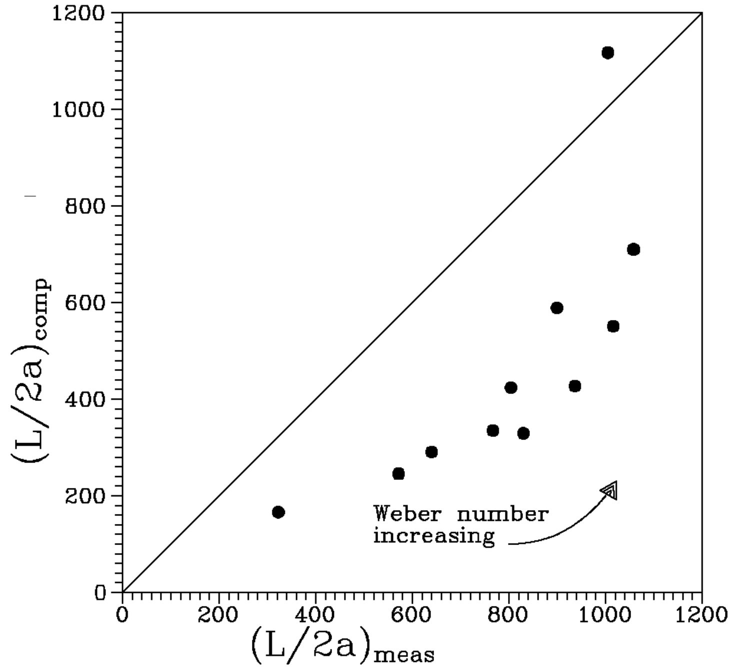
$$L/2a = C_1 \frac{U}{2a\alpha^*} = C_1 \sqrt{We}(1+3Z)$$

$$C_1 = \frac{C_2}{1+3El/(1+3Z)^2}\,; \quad C_2 \approx 12$$

α^* from the present theory

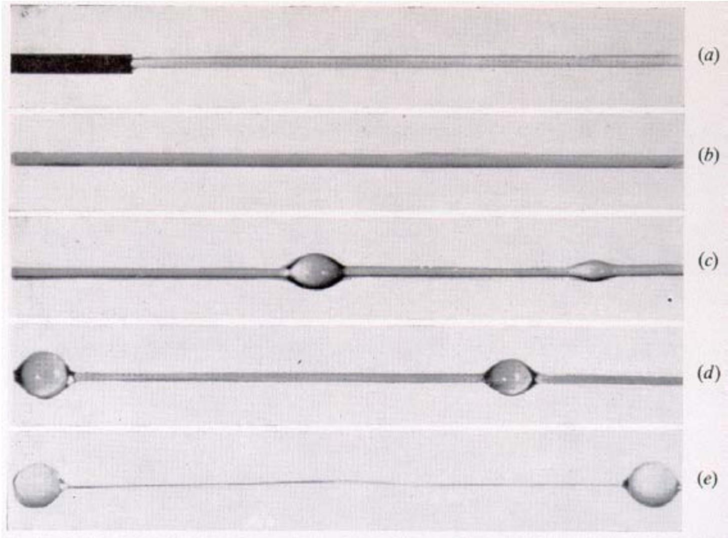
Break-up length by Kroesser and Middleman (1969) and from present theory using coefficients C_1 from Kroesser and Middleman

Validation of the Theory - III



$$L/2a = C_1 \frac{U}{2a\alpha^*} \quad ; \quad C_1 = 12$$

$$9 \le We \le 407$$



Photograph by Goldin et al., JFM 38 (1969)

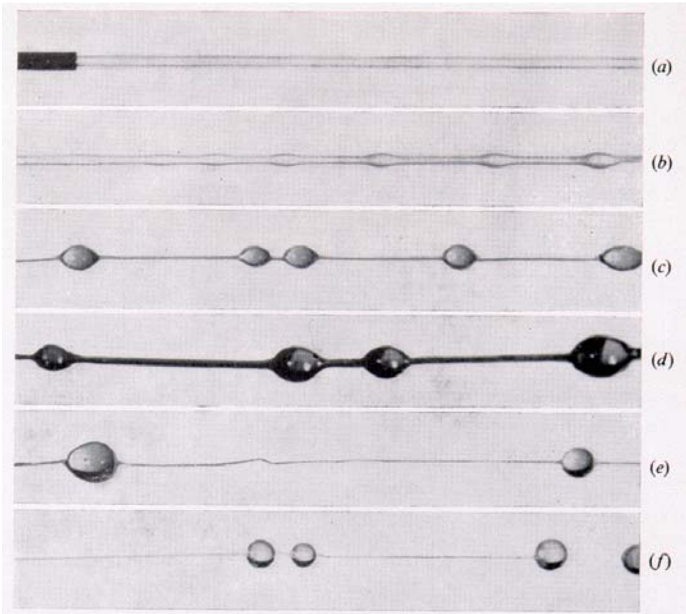
Linear calculation underestimates the break-up length. Strong nonlinear retardation in the late stages

Don‘t calculate break-up lengths with the linear theory

Break-up Mechanisms

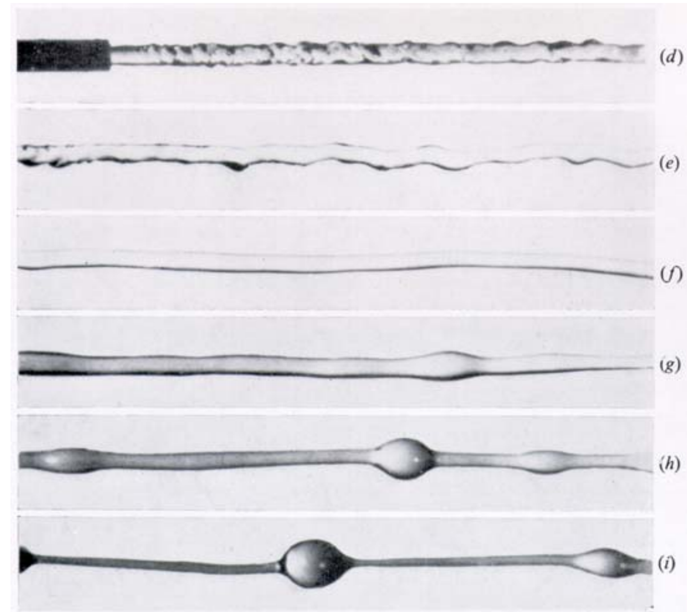
Symmetric deformations

We = 50.2, 500ppm Separan



Antimetric deformations

We =295, 500ppm Separan



Bending instability (antimetric deformations; Yarin 1993)

$$\gamma^2 + \frac{3}{4}\frac{\chi^4}{\rho a^2}\frac{\mu}{1+\gamma\lambda_1}\gamma + \chi^2\left(\frac{\sigma}{\rho a^3} - \frac{\rho_G}{\rho}\frac{\overline{U}_R^2}{a^2} + \frac{T_{zz}}{\rho a^2}\right) = 0$$

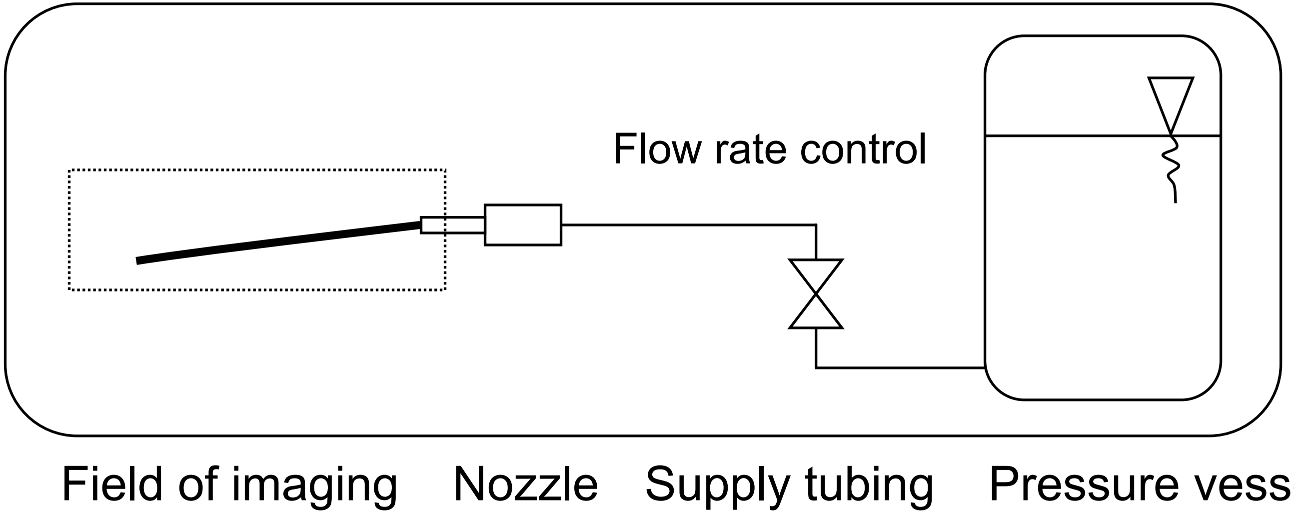
Onset of bending instability

$$\gamma > 0 \quad \Leftrightarrow \quad We > \frac{\rho}{\rho_G}\left(1+\frac{T_{zz}a}{\sigma}\right); \quad \Rightarrow We_{ons} = O\left(10^3\right)$$

(Photographs by Goldin et al., JFM 38 (1969), pp. 689-711)

Break-up Mechanisms for viscoelastic jets

Experimental setup used for investigation of viscoelastic jet breakup mechanisms - similar to A. Haenlein's apparatus from 1931



Nozzle: L/d = 10, d = 0.5, 1, and 2 mm
Liquids: Water and aqueous PAM solutions in three concentrations
Flow rates: 2.6 - 360 l/h

Elongational Rheometer ⇨ Theoretical basics

Cylindrical liquid thread of infinite length

Viscous — Elastic

Capillary forces

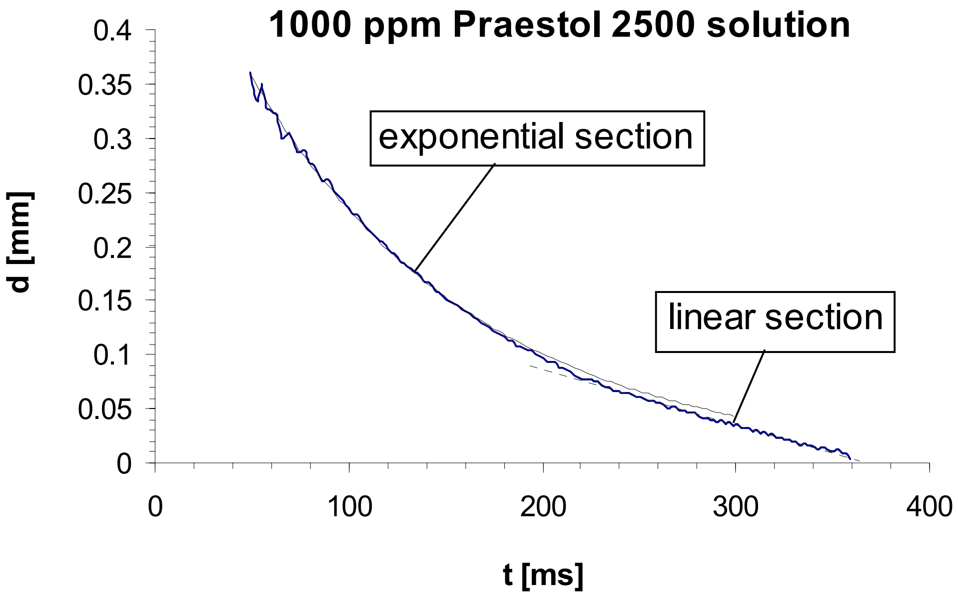
	Thread diameter	Elongational viscosity	Stretching rate	Deborah number
Viscoelastic	$d = d_0 e^{-t/3\Theta}$	$\mu_{el} = \frac{3\sigma\Theta}{d}$	$\dot{\varepsilon} = \frac{2}{3\,\Theta}$	$De = \frac{2}{3}$
Newtonian	$d = d_0 - \frac{\sigma}{\mu_{el}} t$	$\mu_{el} = 3\mu$	$\dot{\varepsilon} = \frac{2\sigma}{3\mu d}$	

Temporal diameter decrease ⟷ Elongational behaviour

Measurement of Elongational Behaviour

Liquid thread

1000 ppm Praestol 2540 solution



viscoelastic behaviour ⟷ relaxation time
→ transient elongational viscosity

quasi-Newtonian behaviour ⟷ steady terminal elongational viscosity

Dimensional Analysis

Relevant parameters → $\mu_{el}, \sigma, \rho, U, d_e, D$

→ Definition of an effective elongational viscosity from the stretching experiments

$$\mu_{el,eff} = k \cdot \mu_{el,exp} \cdot De_d \quad \text{where} \quad \mu_{el,exp} = 3\Theta\sigma / d_e \quad \text{and} \quad De_d = \Theta \frac{U}{d_e}$$

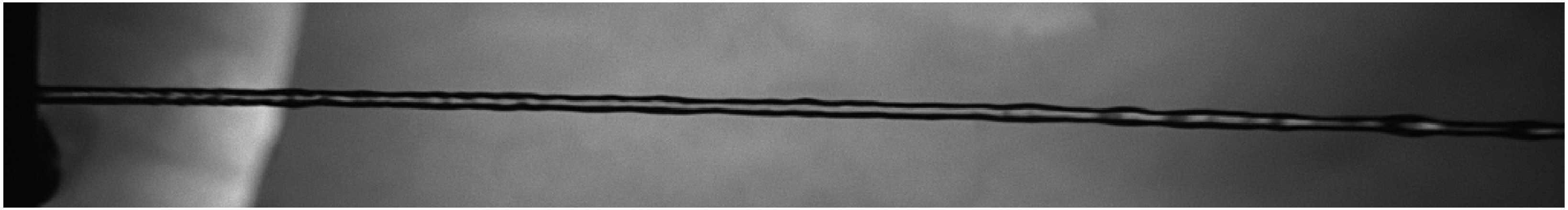
k - empirical parameter to represent results of water and polymer solutions universally

- characteristic for applied flexible/(semi-rigid) and rigid polymers

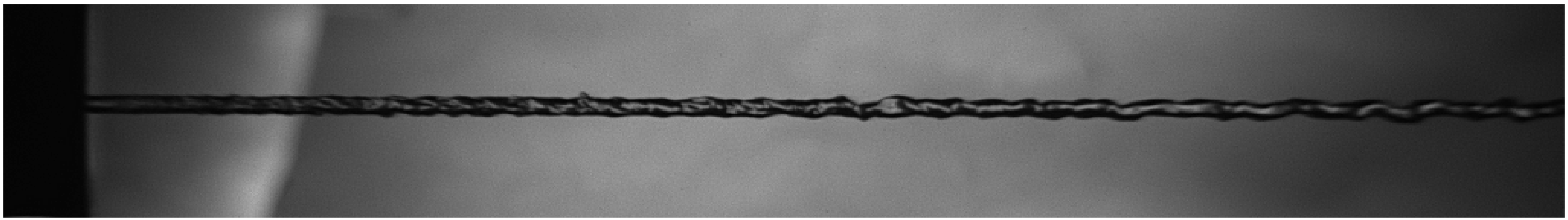
→

$$\frac{k_{flexible}}{k_{rigid}} = \frac{\left(d\mu_{el,t} / d\Theta\right)_{flexible}}{\left(d\mu_{el,t} / d\Theta\right)_{rigid}}$$

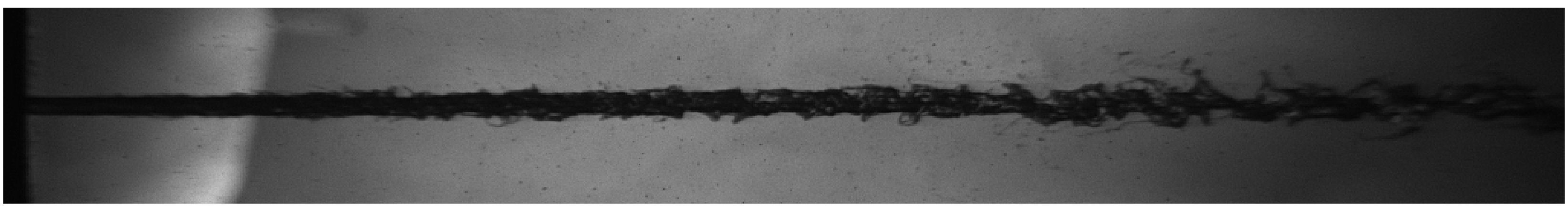
Visualisation of Jet Break-up Mechanisms



100ppm P2500, V=13 l/h, h_{el}=22.5mPas, Re=204, We=298



50ppm P2500, V=64 l/h, h_{el}=11.5mPas, Re=1971, We=7216



50ppm P2500, V=162 l/h, h_{el}=7.2mPas, Re=7962, We=46236

Nozzle d =2mm, aqueous solutions

Direction of flow

Generalised Ohnesorge Jet Break-up Nomogram

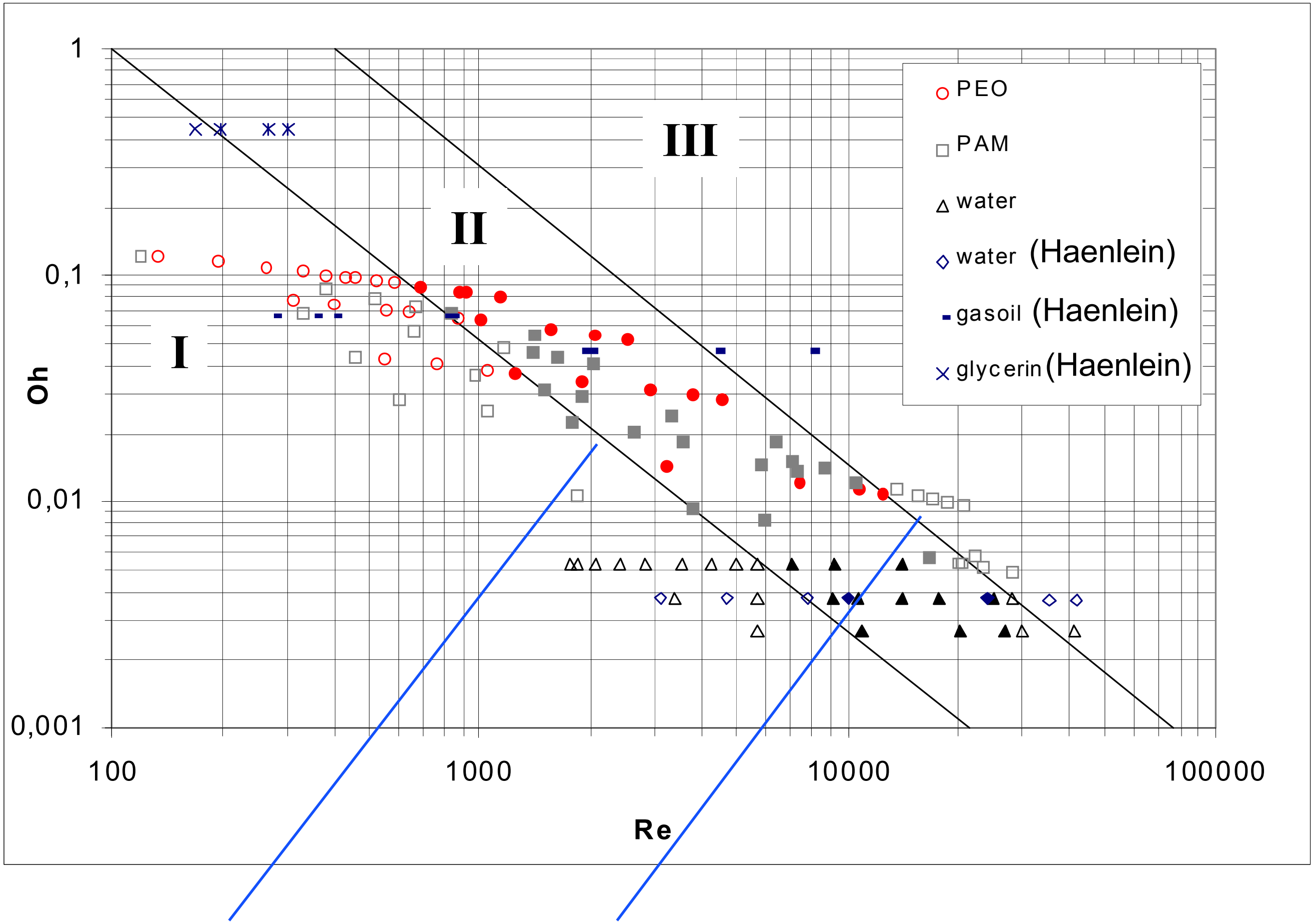
Nomogram for flexible polymers and Newtonians

$$Oh = \eta_{el} / \sqrt{\sigma d \rho}$$

$$\mathrm{Re} = U d \rho / \eta_{el}$$

$$\eta_{el} = k \cdot 3 \sigma \theta / d$$

Symbols:
open - I and III
filled - II



$$Oh = 371.7 \cdot \mathrm{Re}^{-1.285} \qquad Oh = 2624.3 \cdot \mathrm{Re}^{-1.314}$$

Summary and Conclusions

- A linearised temporal stability analysis of viscoelastic jets has been presented
- The dispersion relation agrees with measured disturbance growth rates at small deformations
- The validity of the jet stability theory is limited by the onset of mechanisms leading to non-axisymmetric deformations and breakup retardation by the polymers
- An experimental survey of the viscoelastic jet break-up mechanisms has resulted in a generalisation of Ohnesorge‘s jet break-up nomogram