Numerical Simulation of Turbulent Flows

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Conex Kick-off Meeting

Sofia, 01/11/2003

Overview

- Turbulent motion characteristics
  - •Unstable flow
  - •Turbulent eddies, length scales
  - •Turbulent Energy spectrum
  - •Kolmogorov‘s theory
- Computation of turbulent flow
  - •Direct Numerical Simulation
  - •Reynolds Averaged Numerical Simulation
  - •Large-Eddy Simulation
- Examples
- Resume

Turbulent Motion Characteristics

Reynolds number :

$$\mathrm{Re} = \frac{inertial \;\; forces}{viscous \;\; forces} \geq \mathrm{Re}_{krit}$$

$\mathrm{Re} \geq \mathrm{Re}_{krit}$: ⇨ **Flow unstable** ⇨ **Transition to turbulent motion**

Turbulent motion : all flow quantities φ fluctuate

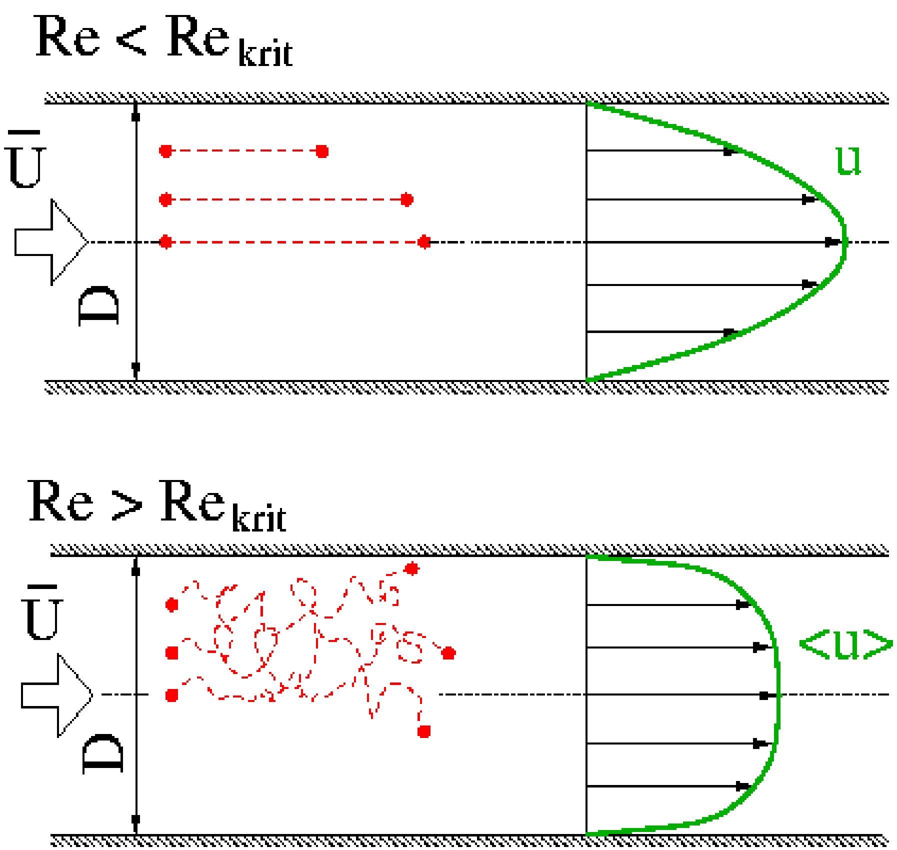
⇨ **motion= unsteady & 3-dimensional !**

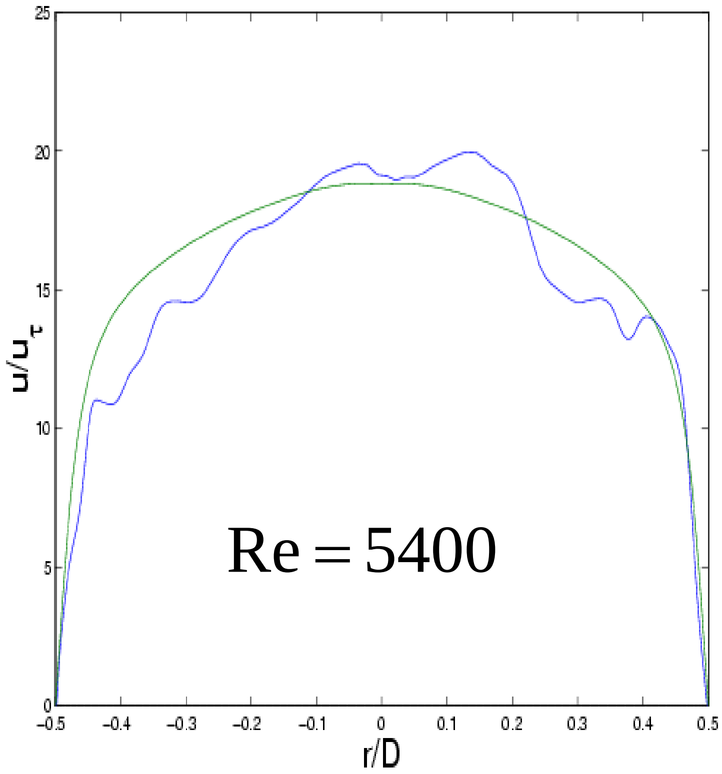
$$\varphi = \varphi \;\; (x_1, x_2, x_3, t)$$

Turbulent Motion Characteristics

Flow through a cylindrical pipe:

$$Re_{krit} \approx 2300$$



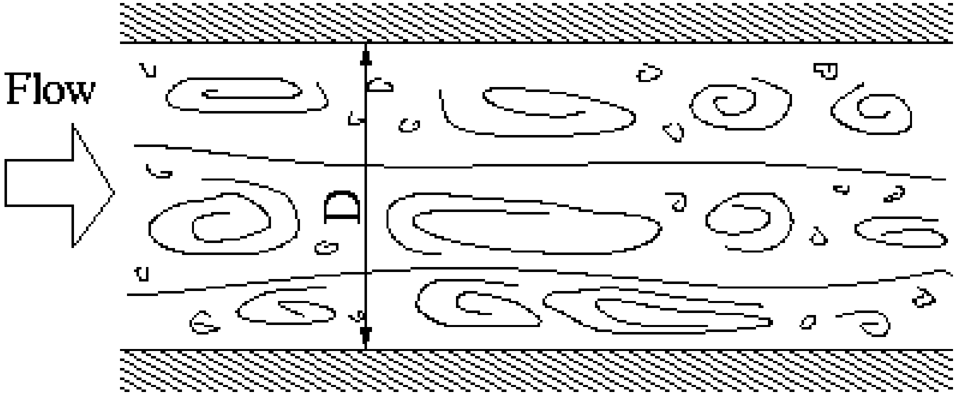


Turbulent Motion Characteristics

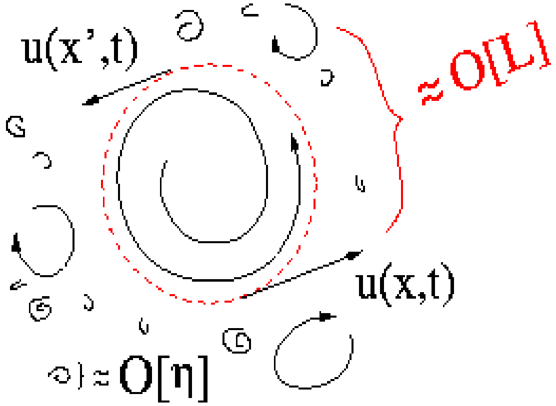
Fluctuating motion characterized by eddies of different sizes:

➡ **turbulent length scales**

Turbulent pipe flow:



turbulent eddies

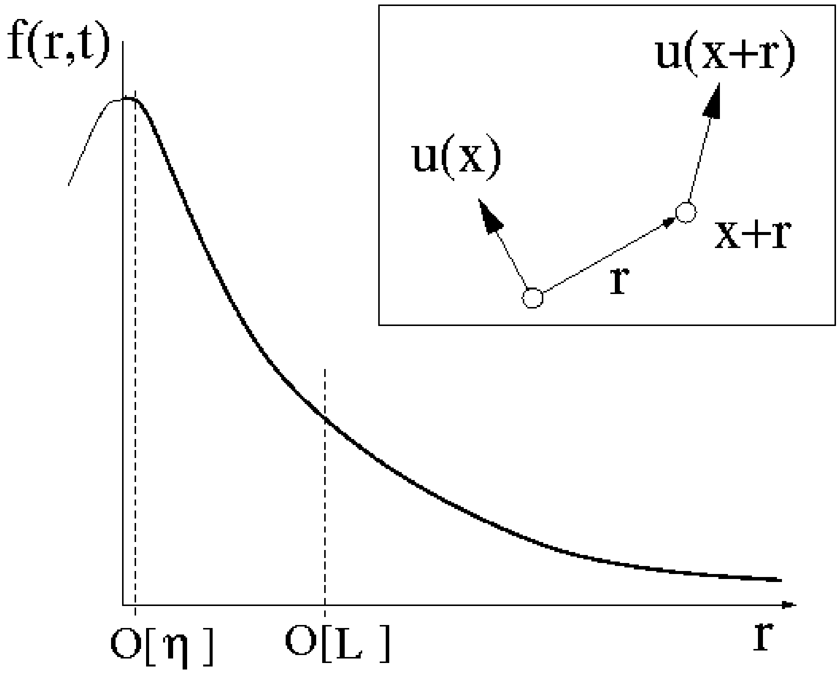


Turbulent Motion Characteristics

Length scales

correlation:

$$f(r,t) = \frac{\langle u(x,t)\, u(x+r,t) \rangle}{\langle u(x,t)^2 \rangle}$$



Integral scale :

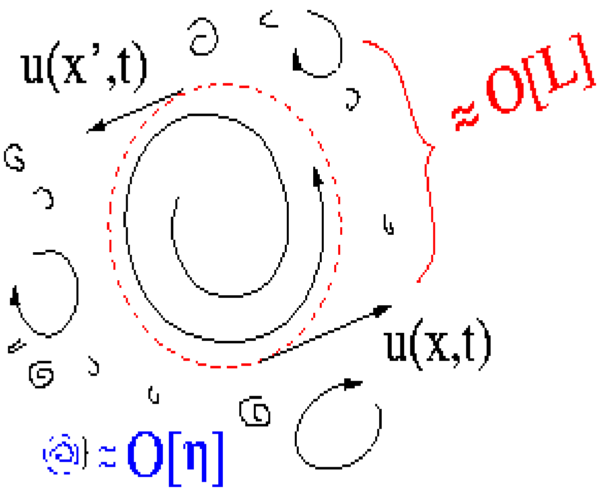
$$L = \int_0^\infty f(r,t)\, dr$$

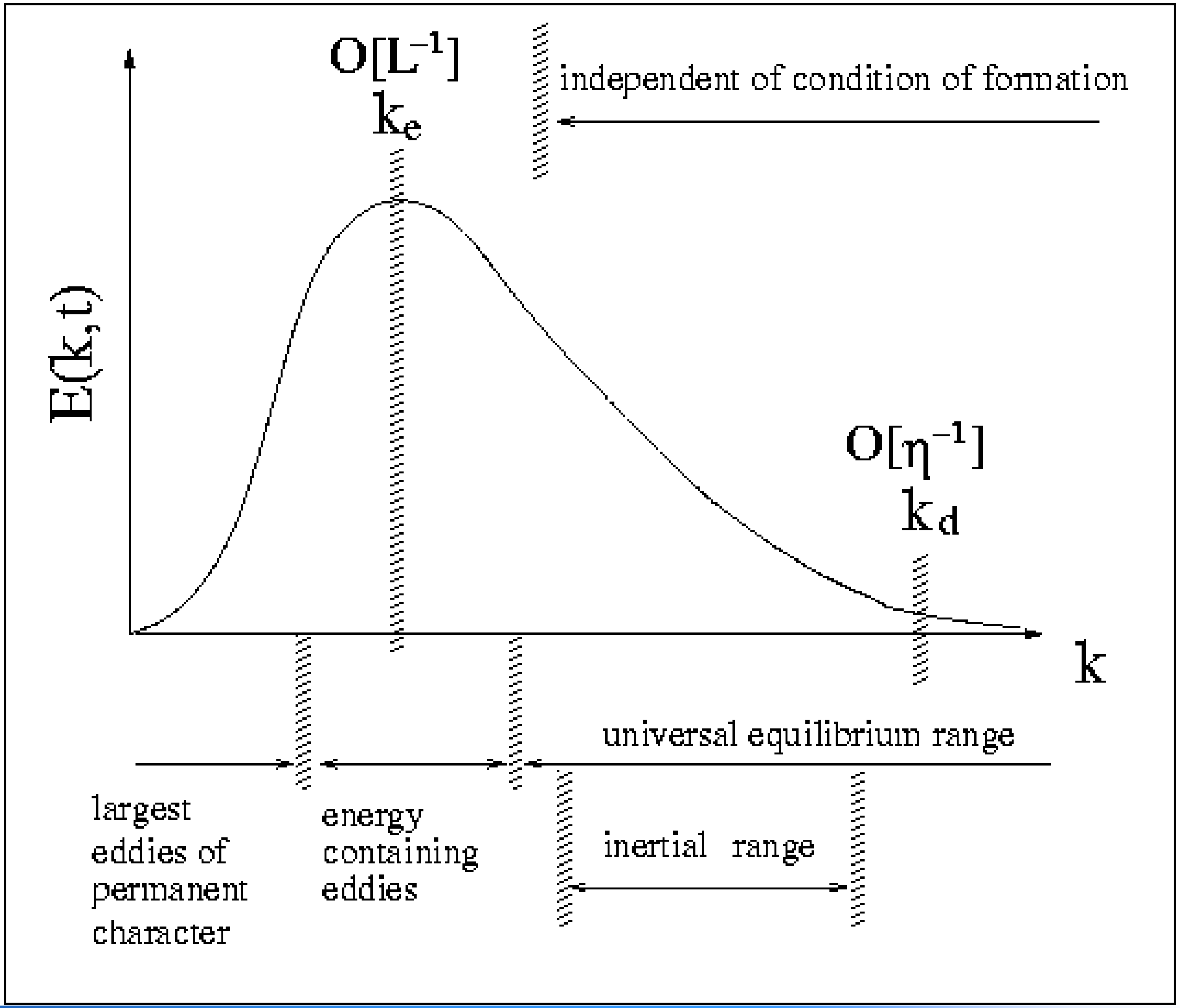
Kolmogorov scale:

$$\eta = \left(\frac{\nu^3}{\varepsilon} \right)^{1/4}, \qquad \varepsilon = \left\langle \nu \frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j} \right\rangle$$

Turbulent Motion Characteristics

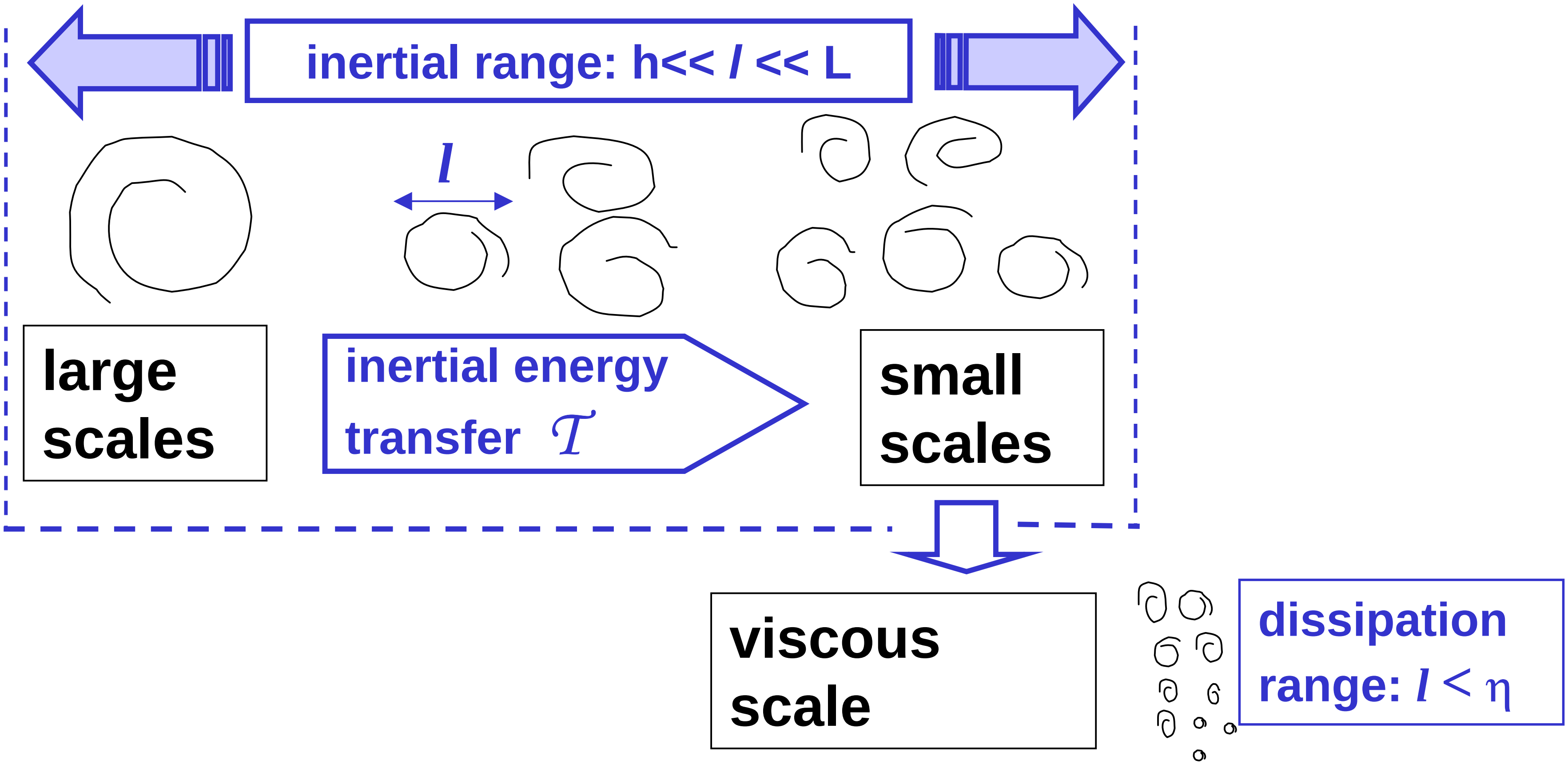
Turbulent Energy Spectrum





Turbulent Motion Characteristics

Kolmogorov‘s eddy cascade hypothesis (1941):



Turbulent Motion Characteristics

Inertial Range: $\eta \ll l \ll L$ **:**

turbulent energy transfer at all length scales l **=1/k depends only on the scale invariant dissipation**

$$\mathcal{T}(l) = \varepsilon = \text{const.}$$

Important for Turbulence Modeling!!

Viscous range: $l < \eta$

dissipation ε **+ viscosity** ν

Computation of turbulent flows

Conservation equations:

Mass:

$$\frac{\partial \rho u_j}{\partial x_j} = 0$$

Momentum:

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \mu \frac{2}{3}\delta_{ij}\frac{\partial u_m}{\partial x_m}\right]$$

Scalar Φ_k :

$$\frac{\partial \rho \Phi_k}{\partial t} + \frac{\partial \rho u_j \Phi_k}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\rho D \frac{\partial \Phi_k}{\partial x_j}\right) + \Omega_k$$

Computation of turbulent flows

Direct Numerical Simulation (DNS)

- **Resolves all relevant scales down to Kolmogorov scale h**
- **no turbulence model needed**
- **provides full turbulence statistics**
- **high resolution and accuracy required ($N_P \sim Re^{9/4}$)**
- **restricted to low Re numbers flow due to exceeding computational cost**

Computation of turbulent flows

Reynolds Averaged Numerical Simulation (RANS)

Statistical approach tracking the (Reynolds) averages over ensemble *N*

$$\langle f \rangle(\vec{x},t) = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} f_n(\vec{x},t) \quad \Rightarrow \langle u_i \rangle, \langle p \rangle, \langle \Phi_k \rangle, \ldots$$

Reynolds decomposition:

$$u_i = \langle u_i \rangle + u_i'$$

$$p = \langle p \rangle + p_k'$$

$$\Phi_k = \langle \Phi_k \rangle + \Phi_k'$$

Computation of turbulent flows

Reynolds averaged equations (case ρ=const.):

$$\frac{\partial \langle u_j \rangle}{\partial x_j} = 0$$

$$\frac{\partial \langle u_i \rangle}{\partial t} + \frac{\partial \langle u_i \rangle \langle u_j \rangle}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \langle p \rangle}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\frac{\mu}{\rho}\frac{\partial \langle u_i \rangle}{\partial x_j} \right) - \boxed{\frac{\partial \langle u_i' u_j' \rangle}{\partial x_j}}$$

$$\frac{\partial \langle \Phi_k \rangle}{\partial t} + \frac{\partial \langle u_j \rangle \langle \Phi_k \rangle}{\partial x_j} = \frac{\partial}{\partial x_j}\left(D \frac{\partial \langle \Phi_k \rangle}{\partial x_j} \right) - \boxed{\frac{\partial \langle u_j' \Phi_k' \rangle}{\partial x_j}} - \boxed{\left\langle \frac{\Omega_k}{\rho} \right\rangle}$$

Computation of turbulent flows

➡ **Model required for unclosed nonlinear terms:**

$$\langle u_i' u_j' \rangle = -\boxed{\nu_t}\left(\frac{\partial\langle u_i\rangle}{\partial x_j} + \frac{\partial\langle u_j\rangle}{\partial x_i}\right) + \frac{\delta_{ij}}{3}\langle u_m' u_m' \rangle$$

$$\langle u_j' \Phi_k' \rangle = -\boxed{D_t}\frac{\partial\langle \Phi_k\rangle}{\partial x_j}$$

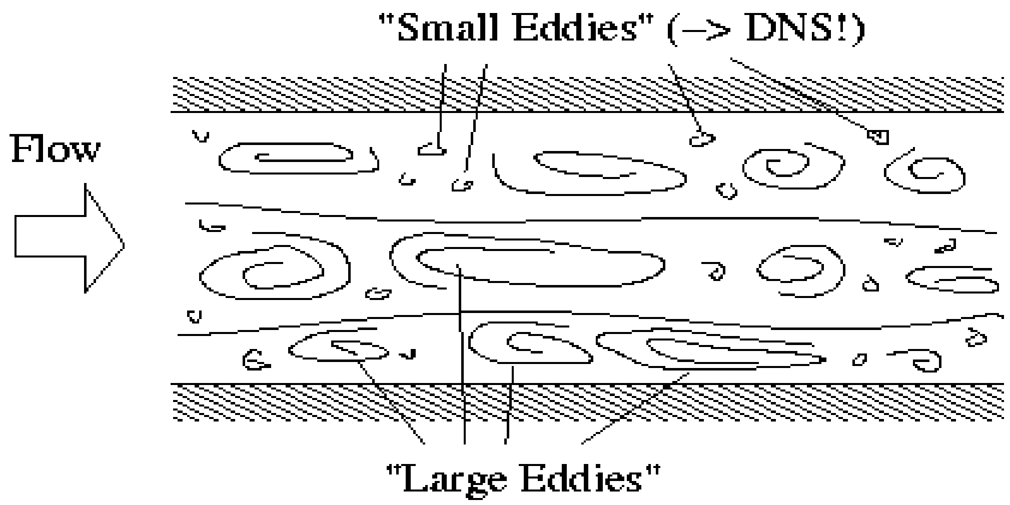
ν_t, D_t **modeled ,e.g., with** $k-\varepsilon$ **- model**

$$\nu_t = C_\mu \frac{k^2}{\varepsilon},\; D_t = \frac{\nu_t}{Sc_t} \quad \textbf{where} \quad k = \frac{1}{2}\langle u_m' u_m' \rangle$$

Computation of turbulent flows

Large Eddy Simulation (LES)

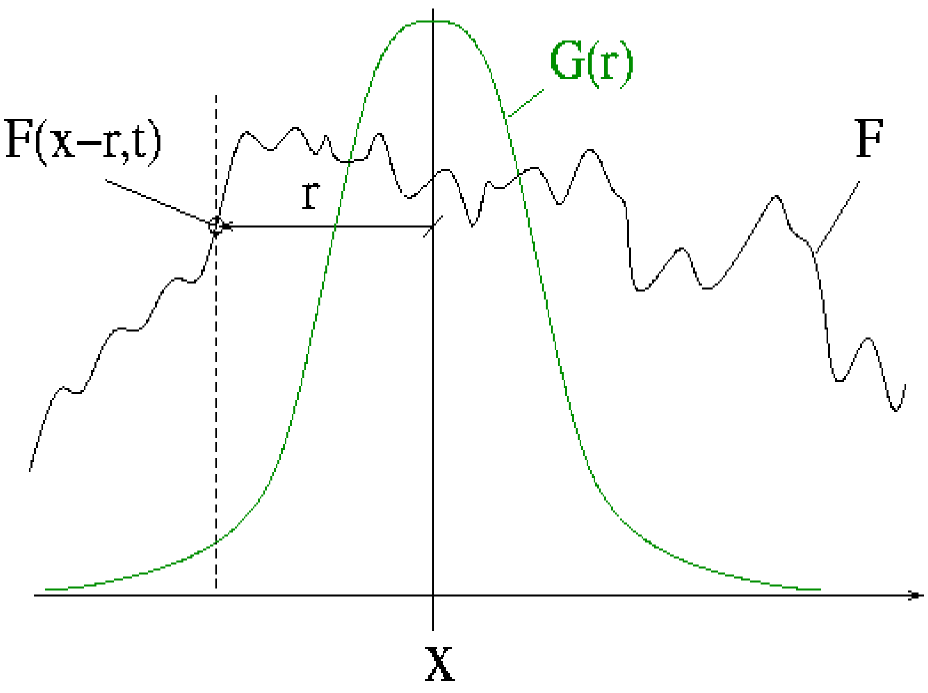
- LES resolves the unsteady 3-dimensional motion of the large structures
- small structures (subgrid-scale) are modeled

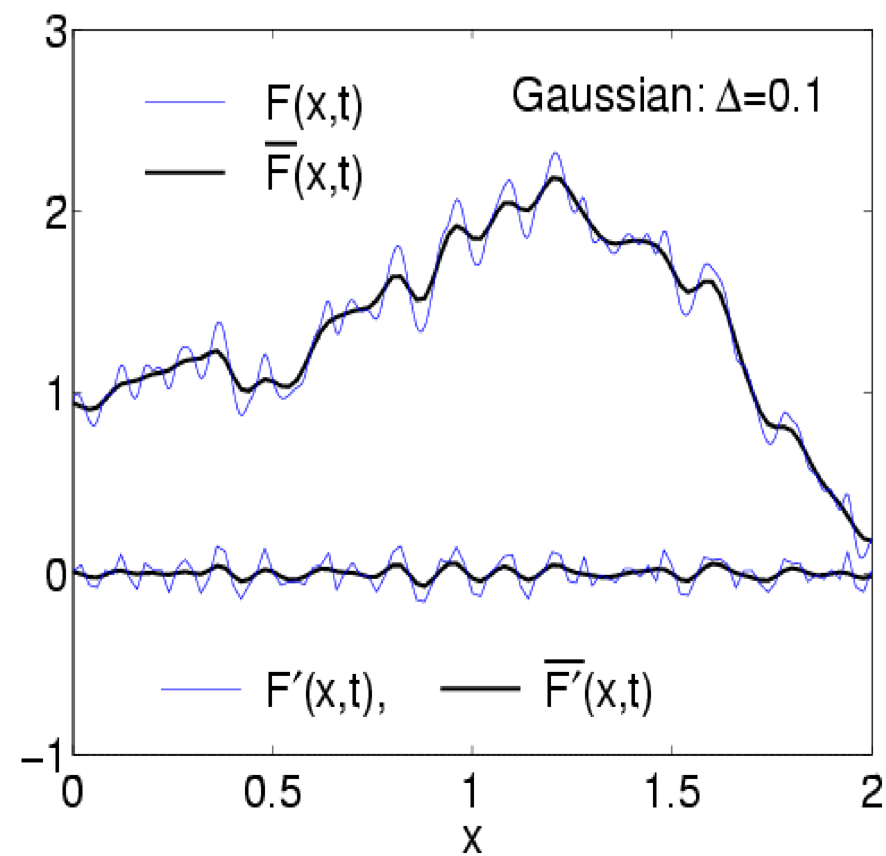


Computation of turbulent flows

Filter operation: ⟹ resolved structures

$$\overline{F}(\vec{x},t) = \int_D F(\vec{x}-\vec{r},t)\,G(\vec{r})\,d\vec{r}$$





Computation of turbulent flows

Filter operation: $\Longrightarrow$ **resolved structures**

➡ **filtered conservation equations (case ρ=const.)**

$$\frac{\partial \overline{u_j}}{\partial x_j} = 0$$

$$\frac{\partial \overline{u_i}}{\partial t} + \frac{\partial \overline{u_i}\,\overline{u_j}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\frac{\mu}{\rho}\frac{\partial \overline{u_i}}{\partial x_j}\right) - \frac{\partial}{\partial x_j}\underbrace{\left(\overline{u_i u_j} - \overline{u_i}\,\overline{u_j}\right)}_{\boxed{\tau_{ij}^R}}$$

$$\frac{\partial \overline{\Phi_k}}{\partial t} + \frac{\partial \overline{u_j}\,\overline{\Phi_k}}{\partial x_j} = \frac{\partial}{\partial x_j}\left(D\frac{\partial \overline{\Phi_k}}{\partial x_j}\right) - \frac{\partial}{\partial x_j}\underbrace{\left(\overline{u_j \Phi_k} - \overline{u_j}\,\overline{\Phi_k}\right)}_{\boxed{\sigma_j^R}} - \boxed{\frac{\overline{\Omega_k}}{\rho}}$$

Computation of turbulent flows

➡ **Model required for unclosed SGS-terms:**

$$\tau_{ij}^{R} - \frac{1}{3}\tau_{kk}^{R}\delta_{ij} = -\boxed{\nu_T}\left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right) = -2\nu_T \overline{S_{ij}}$$

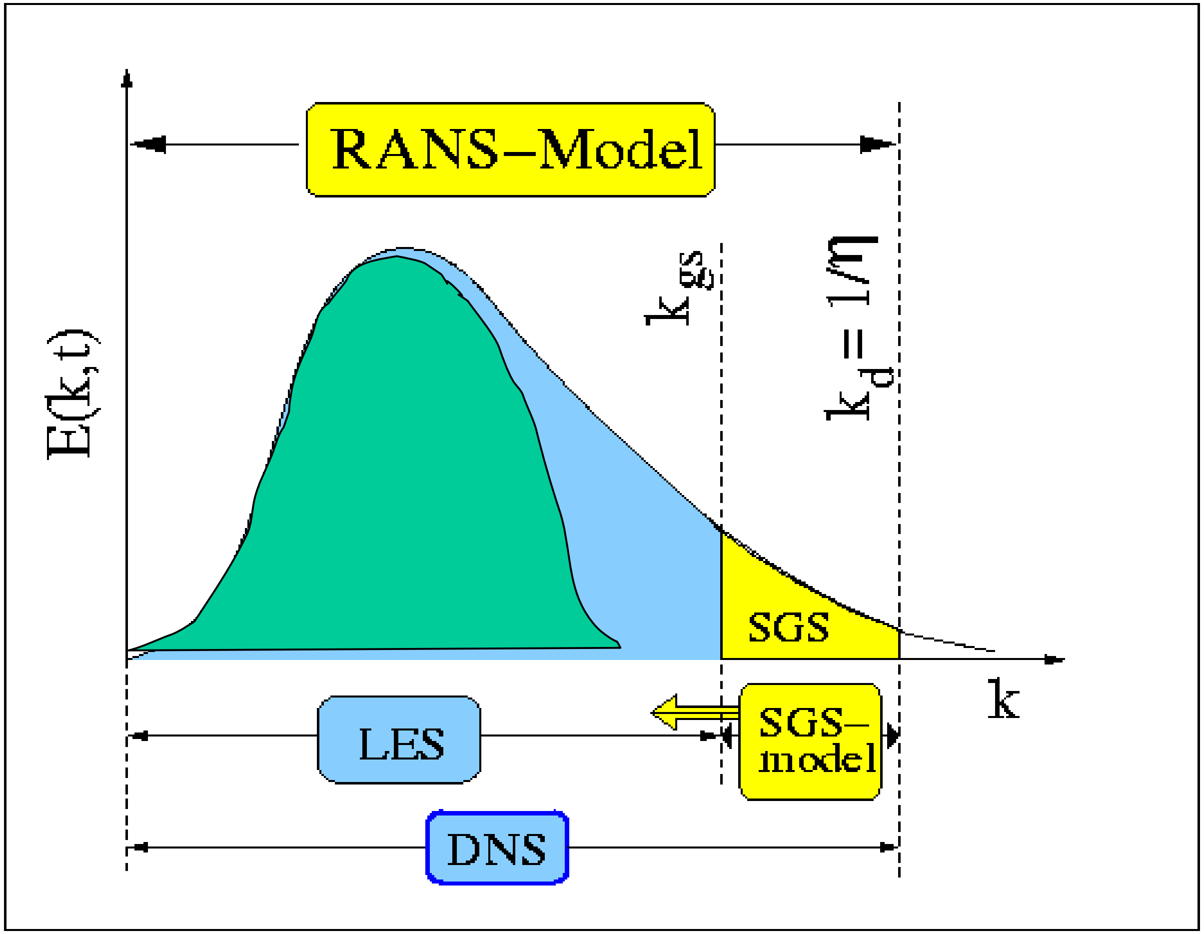
$$\sigma_j^R = -\boxed{D_T}\,\frac{\partial \overline{\Phi_k}}{\partial x_j}$$

ν_T, D_T **modeled ,e.g., with Smagorinsky-type models**

$$\nu_T = (C_S \Delta)^2 \left|\overline{S_{ij}}\right|, \quad D_T = \frac{\nu_T}{Sc_T}$$

Computation of turbulent flows

Simulation methods in wave number space $k = l^{-1}$



Most standard models based on scale invariance of ε

Requires broad inertial range associated with Re>>1

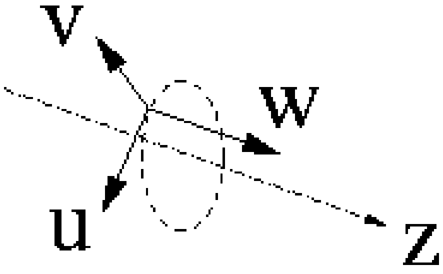
Problem at low Re !

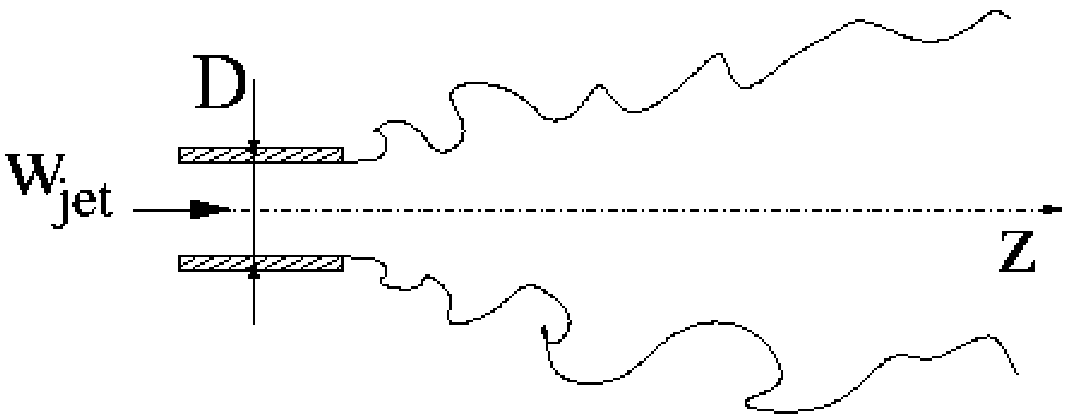
Examples

DNS of a turbulent cylindrical jet

$$\mathrm{Re} = \frac{w_{jet} D}{\nu} = 2400$$

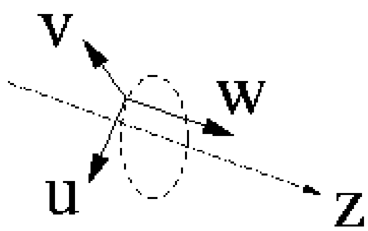
Grid: $N_P \approx 1.2 \cdot 10^6$

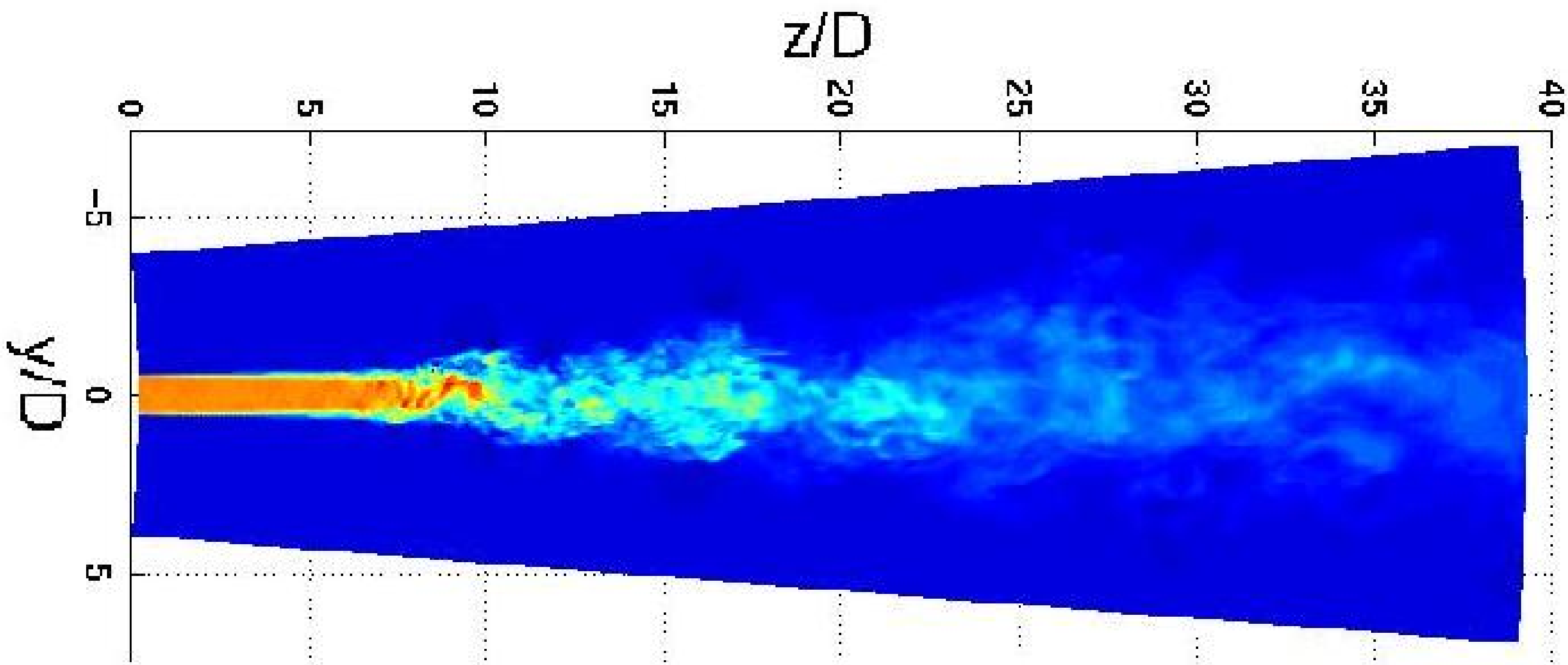


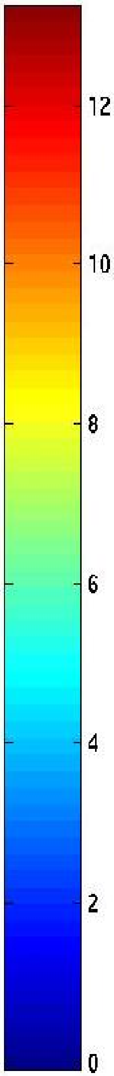


Examples *DNS of a turbulent cylindrical jet*

Axial velocity w/u_0

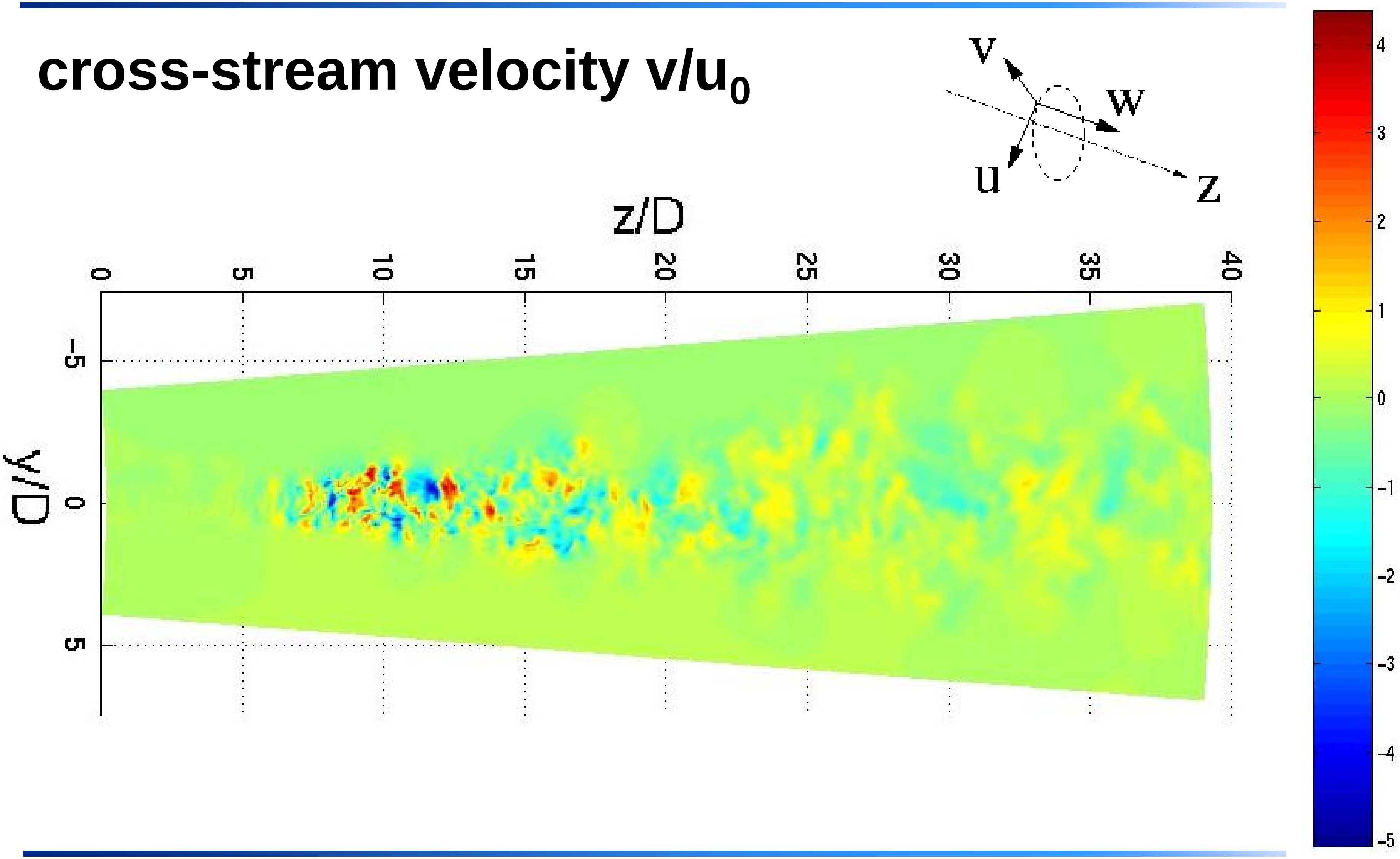






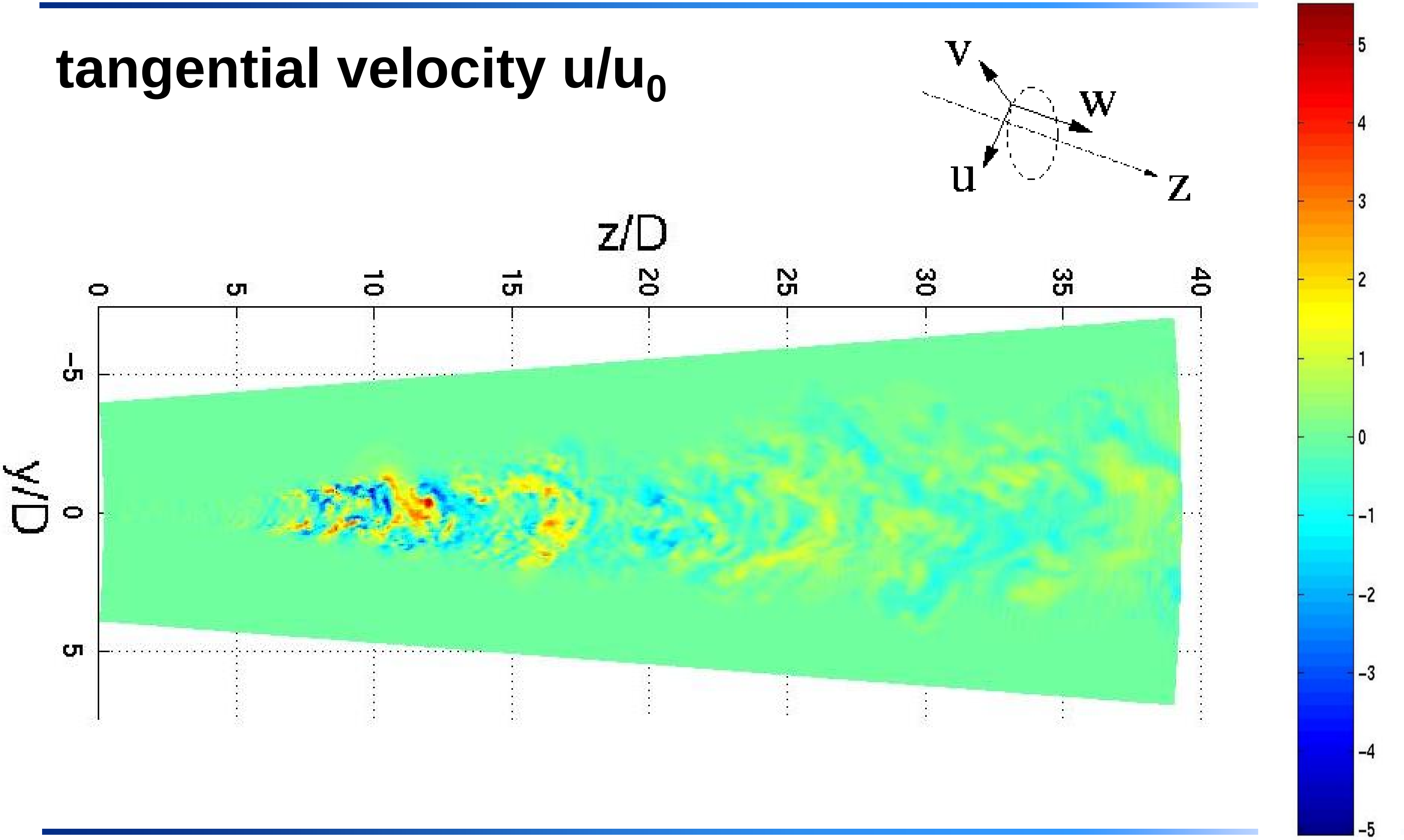
Examples *DNS of a turbulent cylindrical jet*

cross-stream velocity v/u_0



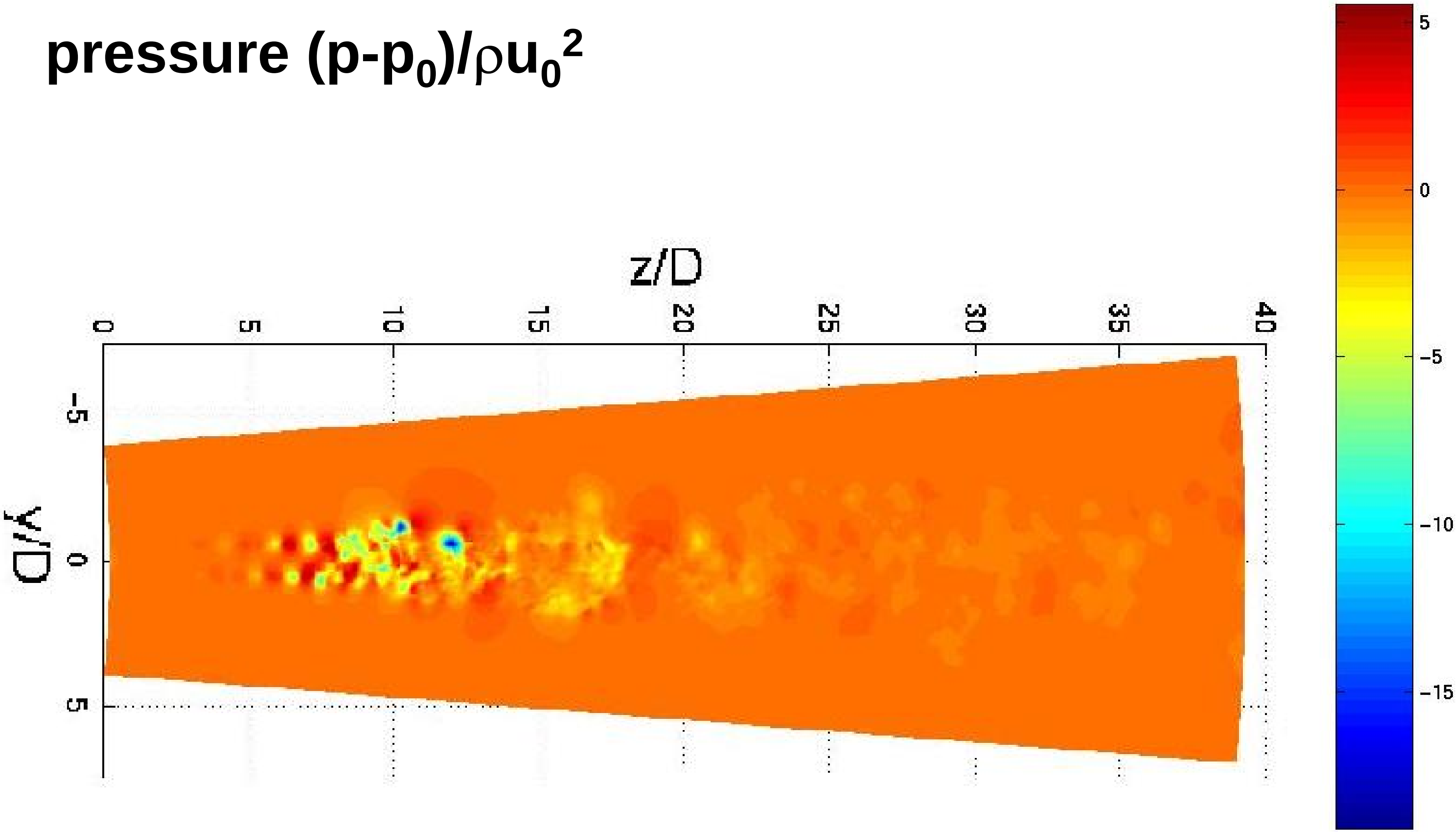
Examples *DNS of a turbulent cylindrical jet*

tangential velocity u/u_0



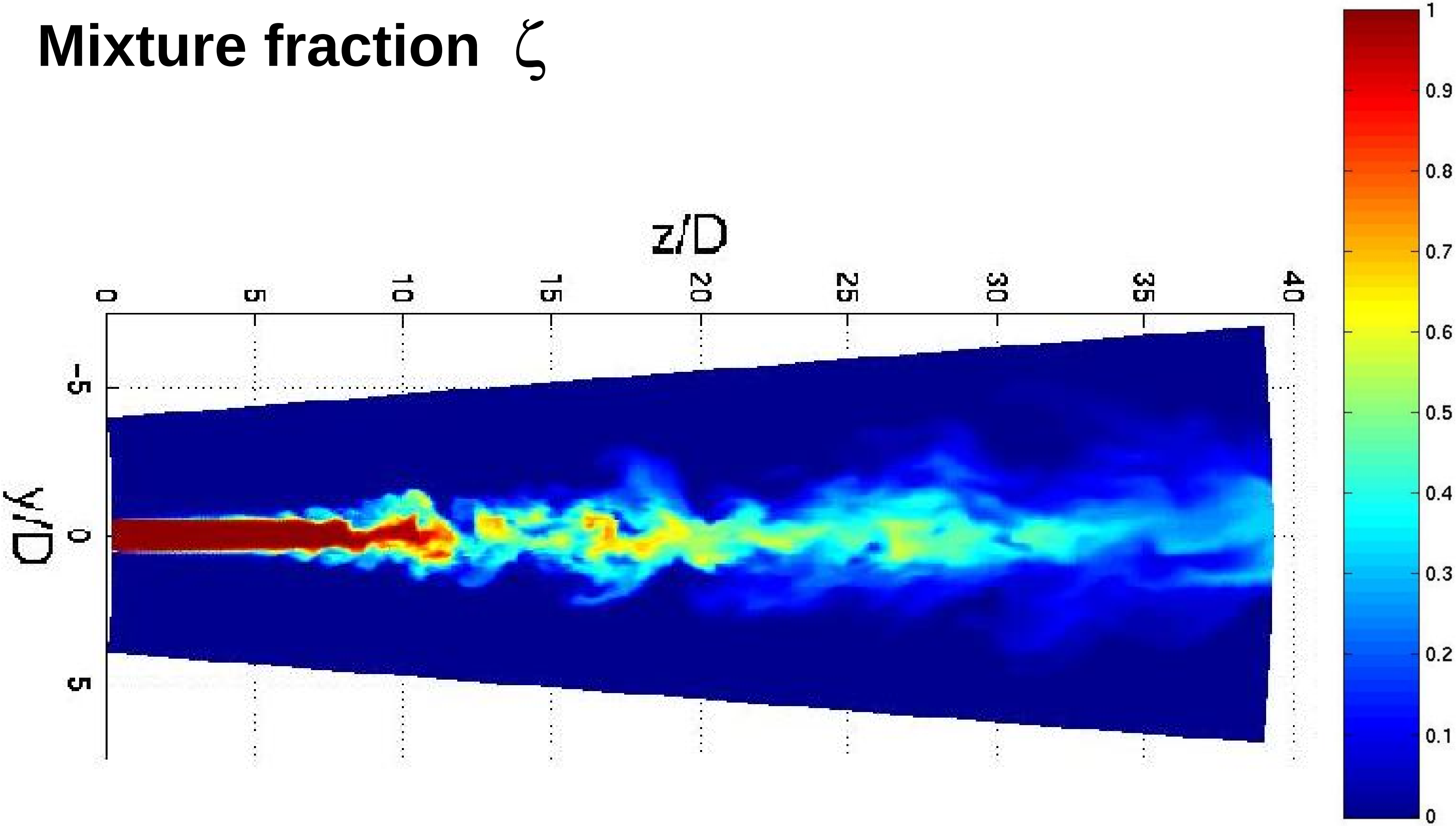
Examples *DNS of a turbulent cylindrical jet*

pressure $(p-p_0)/\rho u_0^2$



Examples *DNS of a turbulent cylindrical jet*

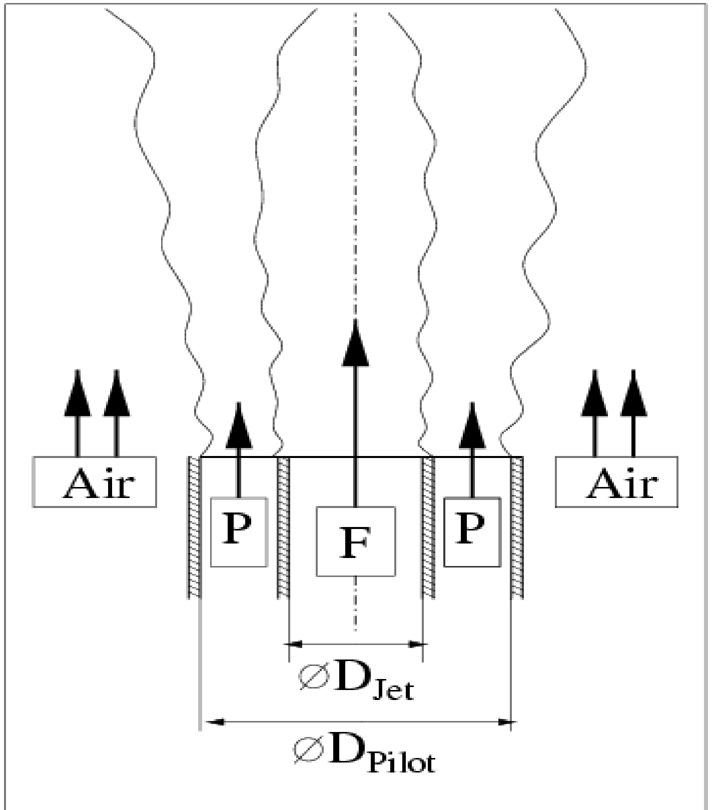
Mixture fraction ζ



LES of a methane-air jet flame (Sandia D flame)

Piloted methane-air diffusion flame

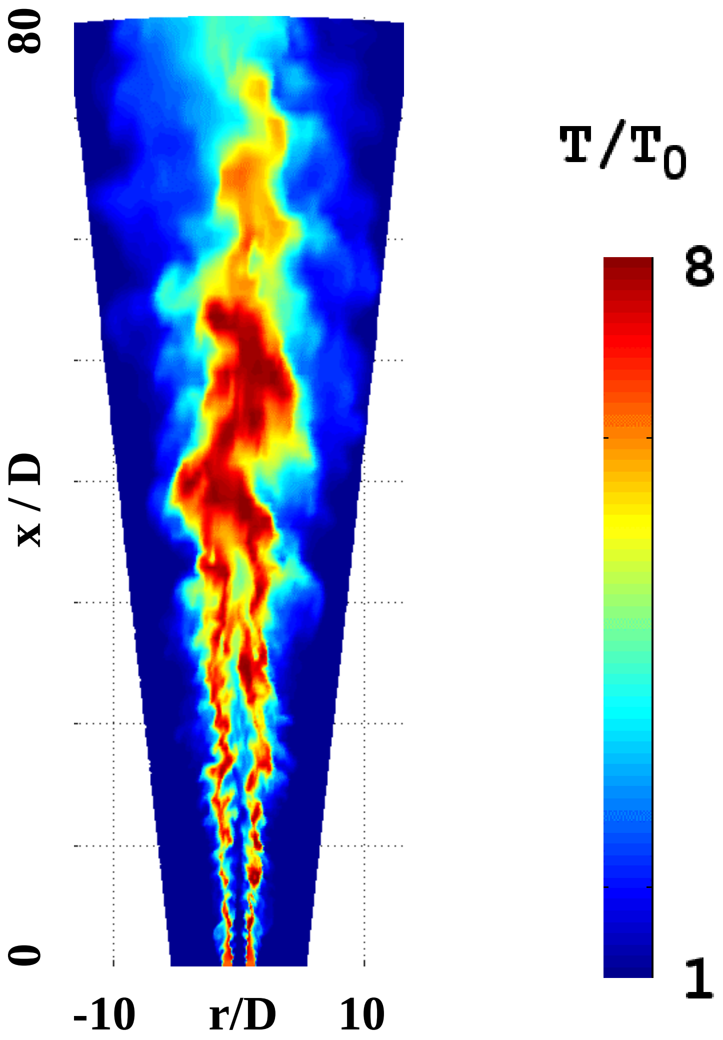
$$\mathrm{Re} = \frac{w_{jet} D_{jet}}{\nu} = 22400$$



Nozzle of the piloted jet

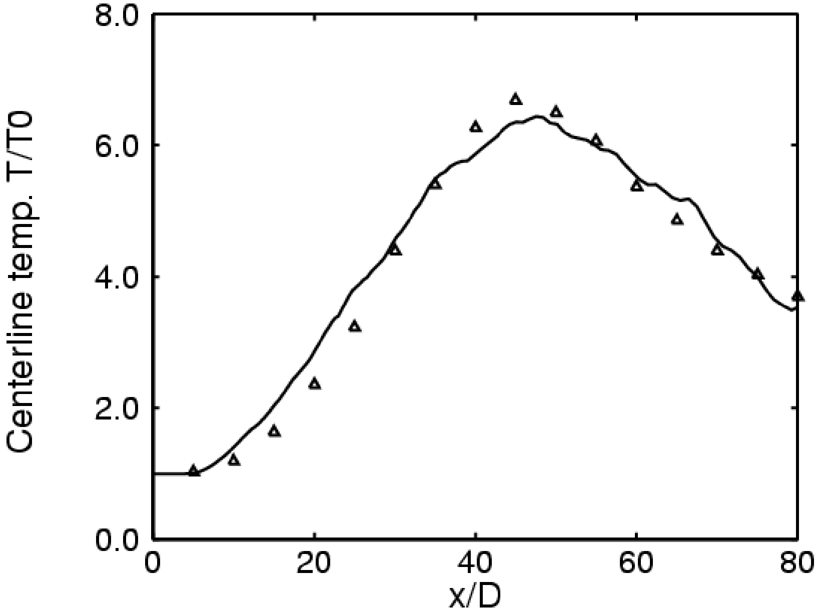
Examples *LES of a turbulent reacting jet*

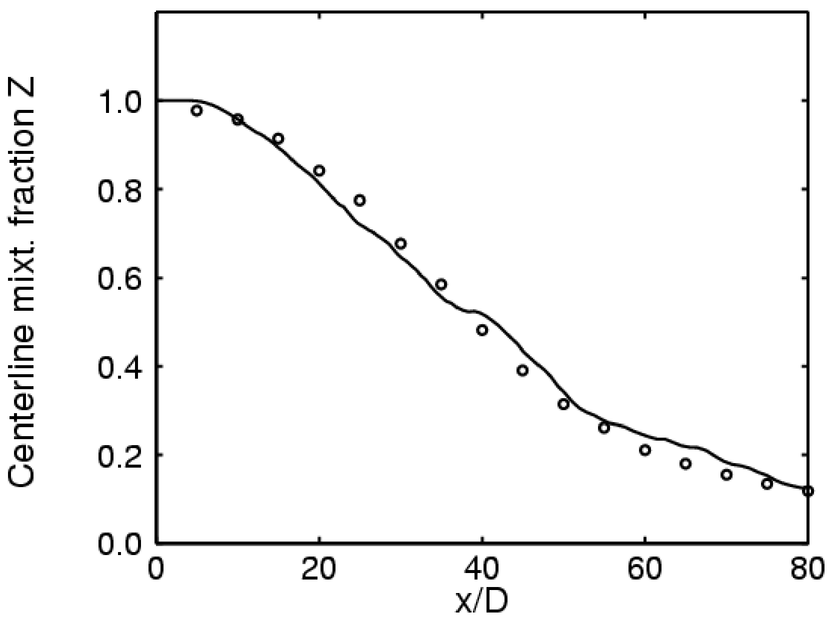
Instantaneous temperature field T/T_0 (T_0 =293K)



Examples *LES of a turbulent reacting jet*

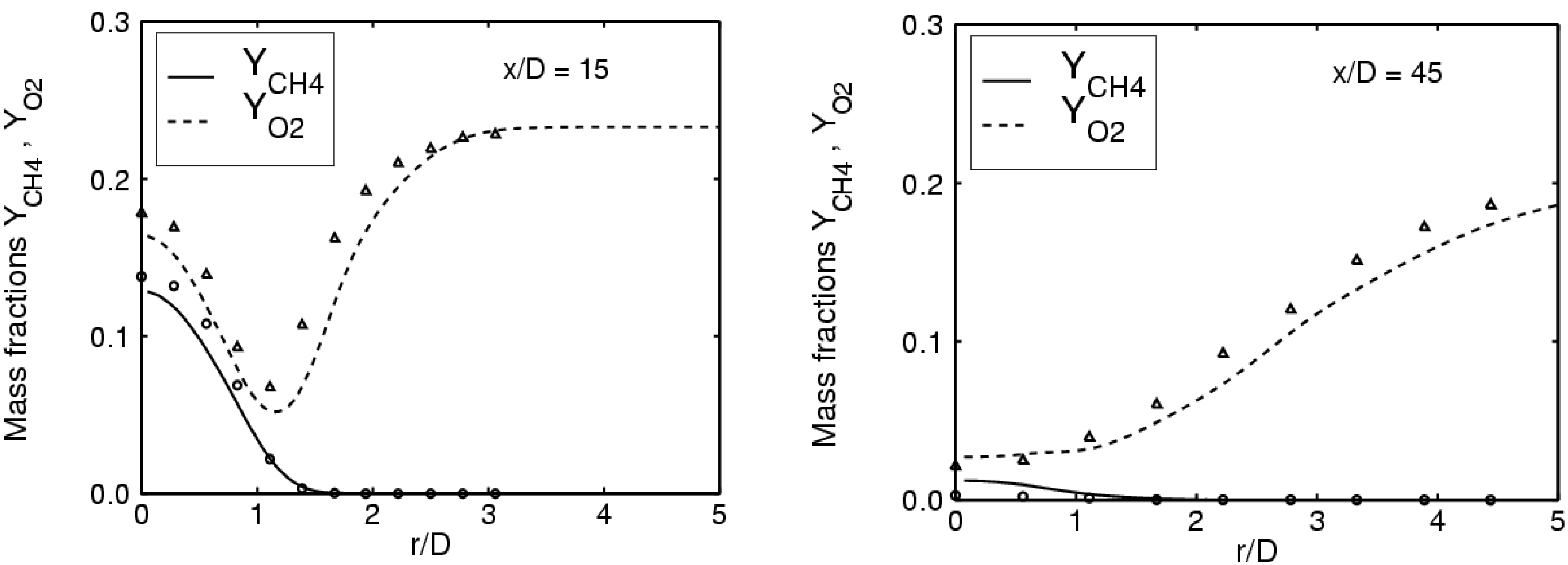
Temperature T/T_0 and mixture fraction Z along centerline of the jet



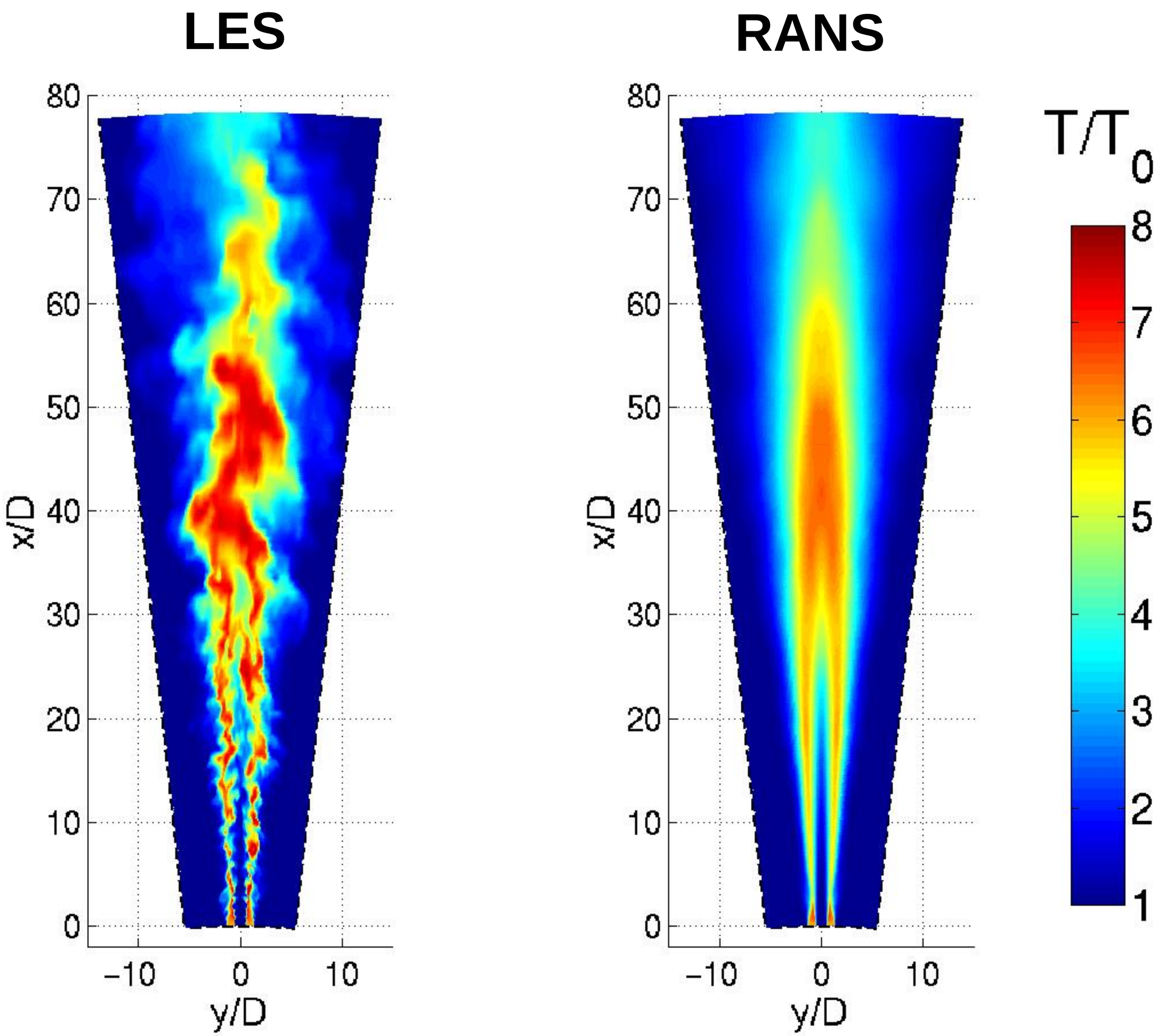


Examples *LES of a turbulent reacting jet*

Radial profiles of mass fractions Y_{CH4} and Y_{O2} at downstream locations x/D=15 and x/D=45

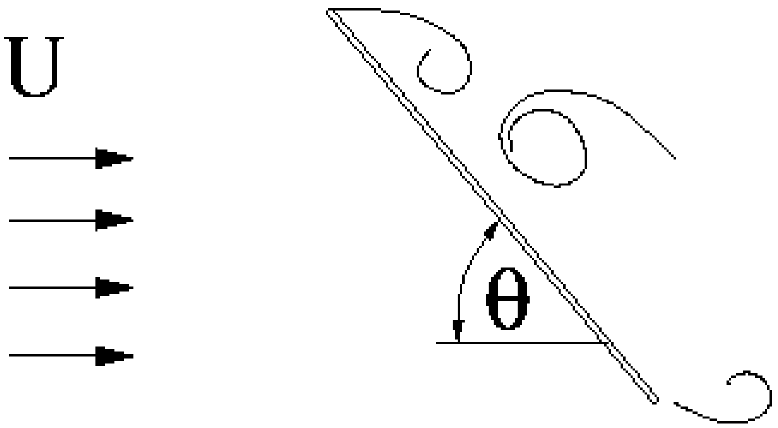


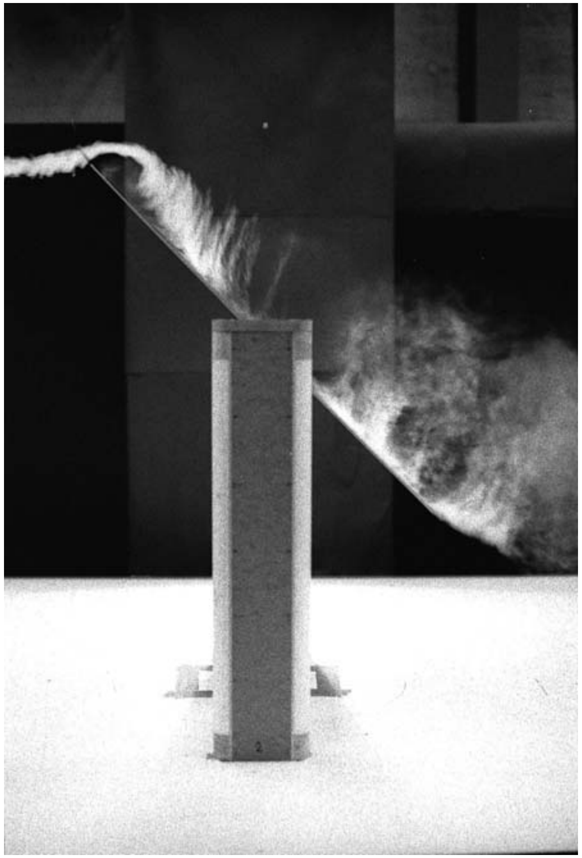
Examples *LES of a turbulent reacting jet : LES↔RANS*



RANS of flow around a flat plate with different angles of attack Θ

$$Re = \frac{U\,L_{plate}}{\nu} = 1.9 \cdot 10^{6}$$

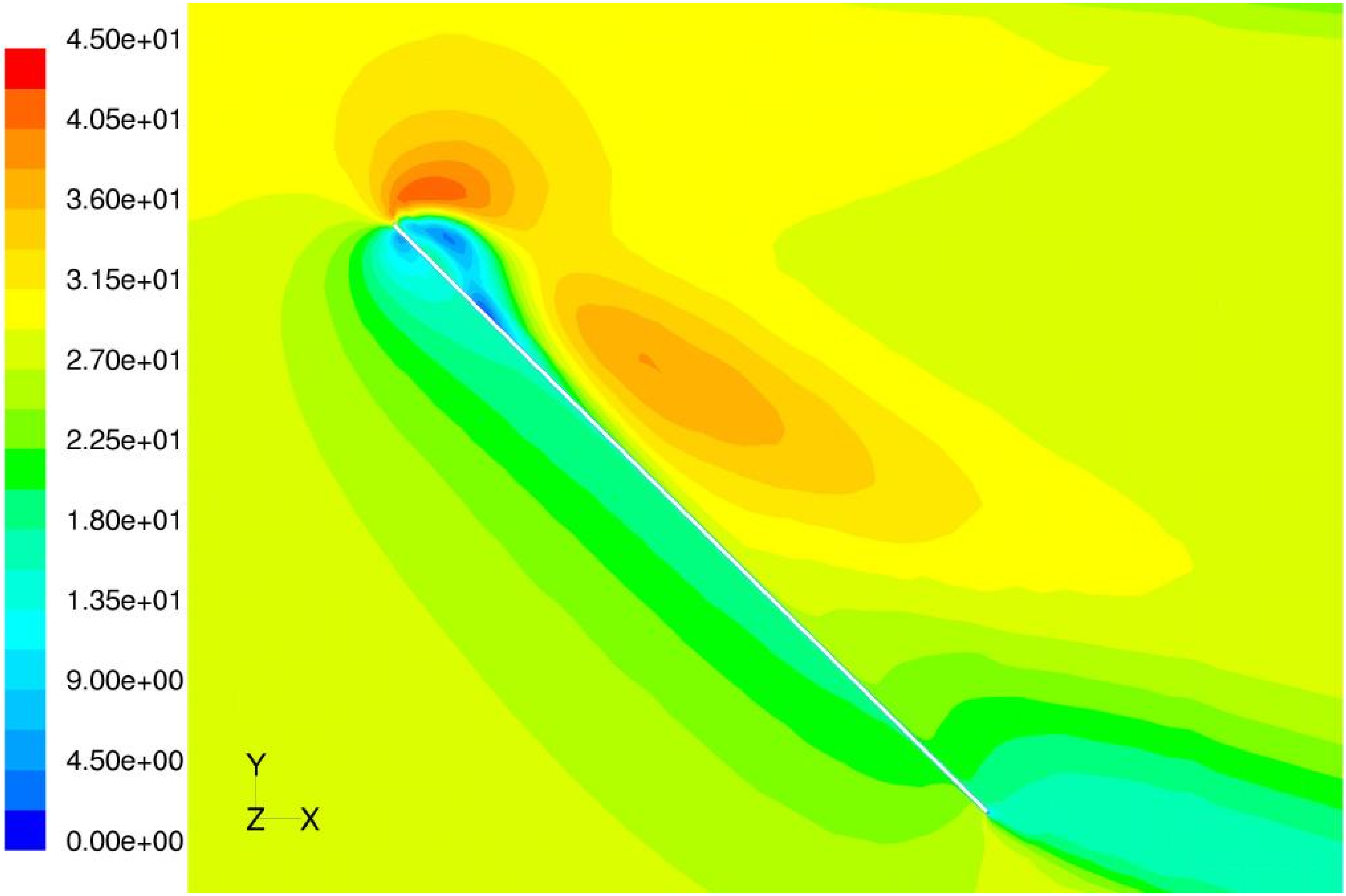




Turbulence model : k-ε (Code: FLUENT)

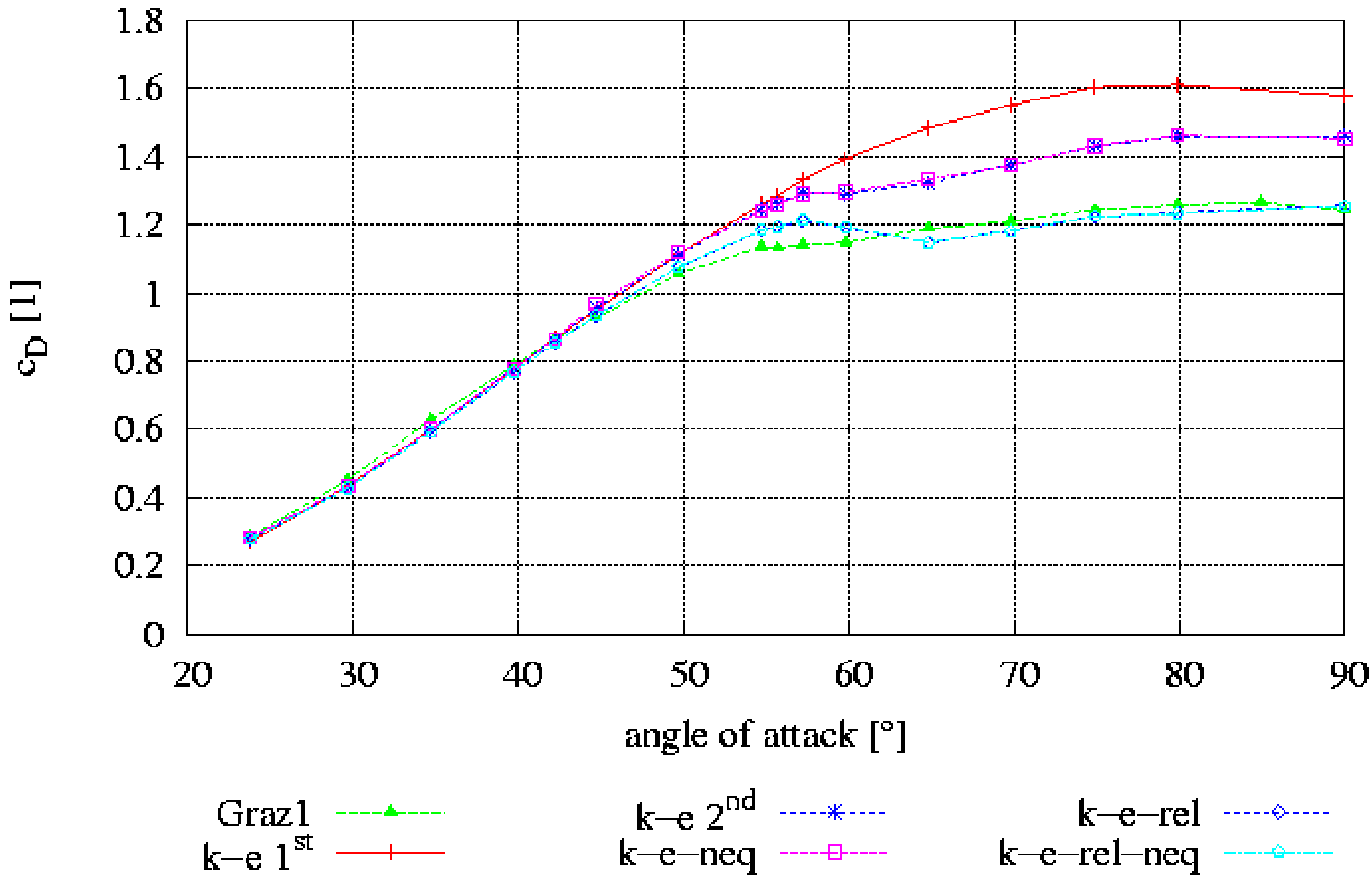
Examples *RANS of flow around a flat plate*

absolute velocity in m/s



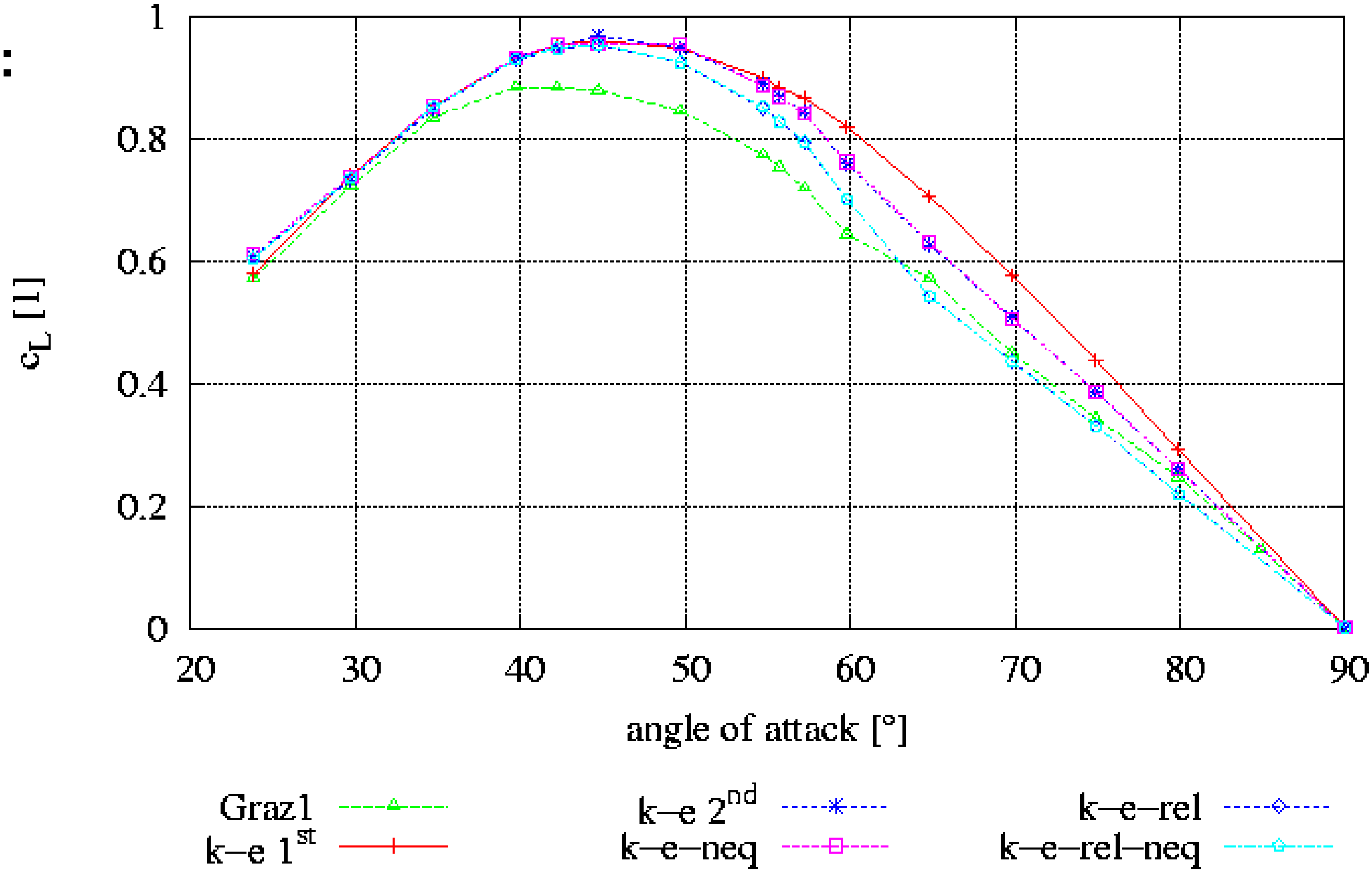
Examples *RANS of flow around a flat plate*

Drag:



Examples *RANS of flow around a flat plate*

Lift:



Resume *Appropriate method?*

DNS?

- **Unfeasible for technically relevant engineering flows**
- **Restricted to scientific applications**

LES?

- **Still computationally too expensive for many technically relevant engineering flows (aerodynamic flows, complicated geometries, etc.)**
- **Useful, when unsteady large scale motion has to captured (mixing process, combustion, etc.)**
- **Great potential for applications in future due to increase of computer power**

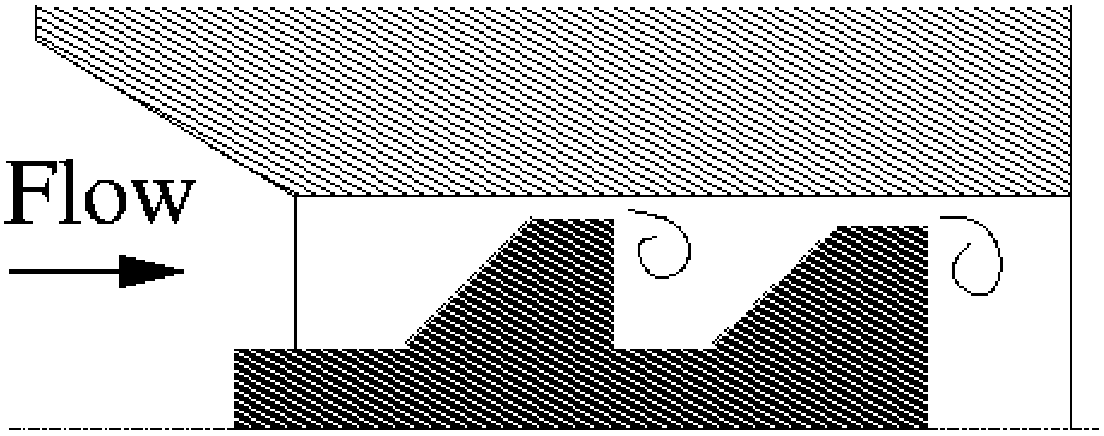
Appropriate method?

RANS?

- **Method of choice in engineering flows**
- **Limited predictive capability of standard turbulence models**
- **More sophisticated turbulence models computationally expensive (but mostly still cheaper than LES !)**

Resume *Appropriate method?*

Homogenizer



Complex geometry

(engineering flow)

RANS with FLUENT

models : standard k-ε, realizable k-ε,

Reynolds-Stress-Transport (?)